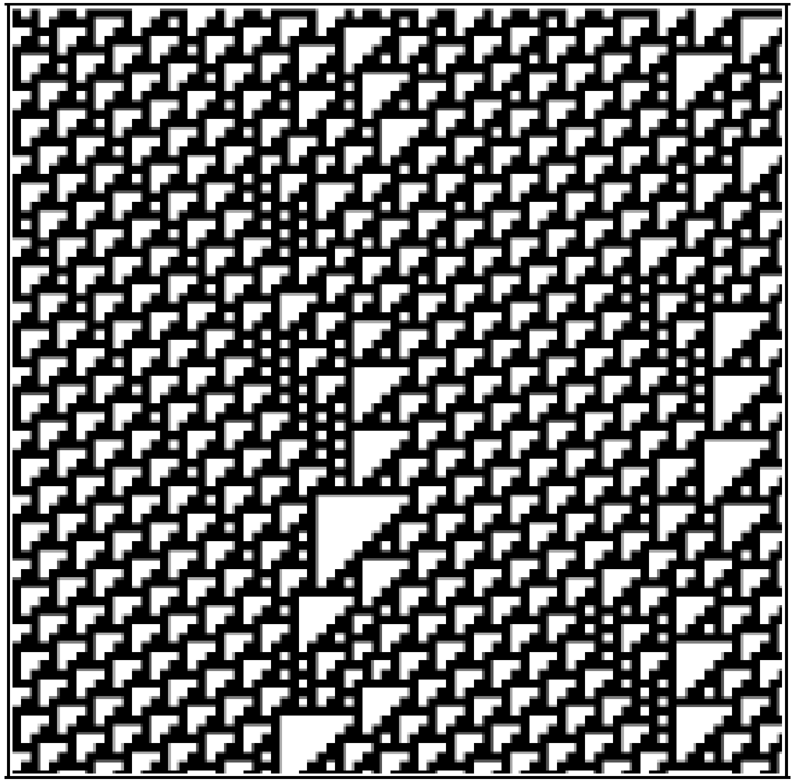

Information transfer by particles in cellular automata

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Information transfer in CAs: Motivation

- Why study information transfer?
  - It's a hot topic, but not adequately quantitatively addressed.
  - Inferred importance in dynamics of systems from many disciplines, e.g. computation, flocking, biological systems, solitons.
  - In particular for artificial life, e.g.:
    - analysis of adaptive systems
    - information-driven evolutionary design (e.g. empowerment).
  - Conjectures about information transfer in phase transitions.

- Why study information transfer in CAs?
  - Powerful models of many real world systems
  - Qualitative nature of information transfer in CAs is established
  - Emergent structures facilitating information transfer have known analogues in real systems

Information transfer by particles in CAs

- Aim: demonstrate how to profile information transfer locally:
  - i.e. as a function of space-time
- Application 1: Demonstrate that particles are the information transfer agents in CAs
- Application 2: Compare information transfer metrics, and study their parameters.

Contents

1. Background:
   1. Information transfer in CAs
   2. Information theory

2. Derive Local Transfer Entropy metric

3. Apply to CAs, proving particles are dominant information transfer agents

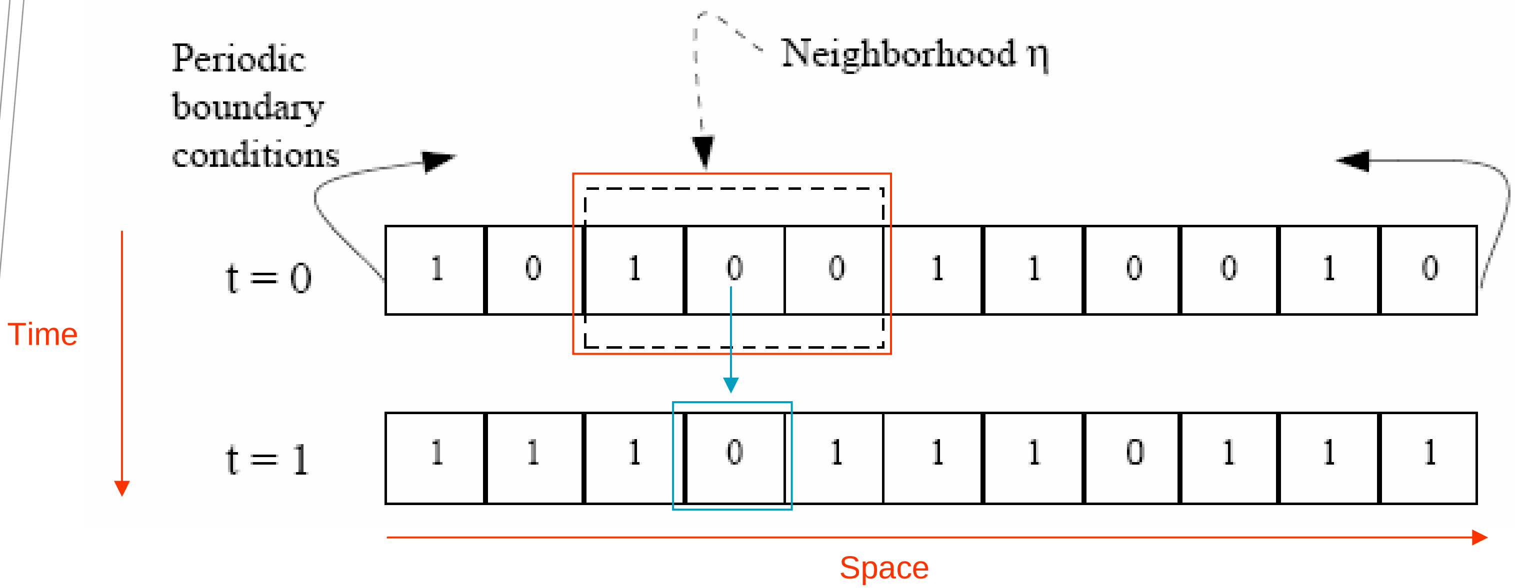
Cellular Automata – micro-level rules

Rule table ϕ:

neighborhood:	000	001	010	011	100	101	110	111
output bit:	0	1	1	1	0	1	1	0

= Rule 0x6e = Rule 110

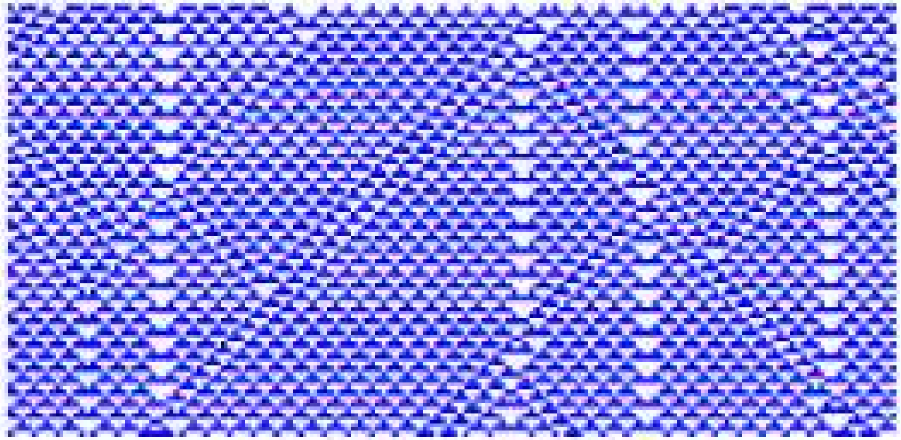
Lattice:



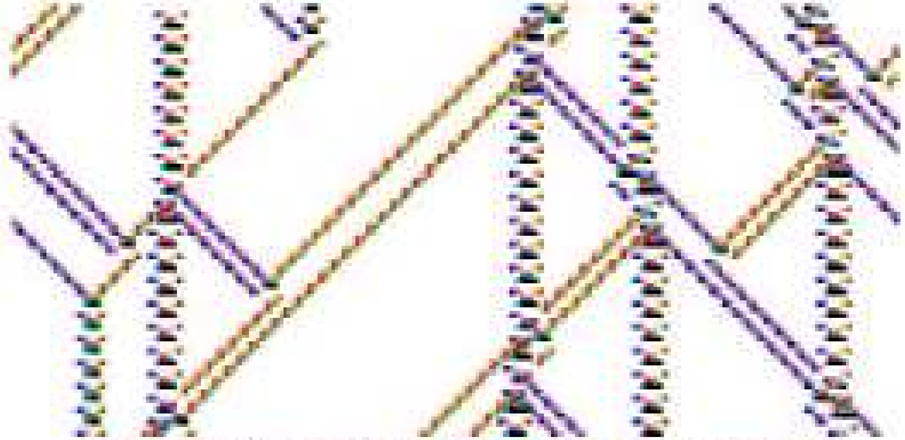
- “Computation in Cellular Automata: A selected review”, Mitchell, 1998

Cellular Automata – emergent structure

- “Classifying Cellular Automata Automatically …”, Wuensche, 1999



cells by value



cells by look-up and filtered

- Emergent structure:
  - Domain
  - Particles
    - Gliders, Domai
  - Collisions

Blinkers

- Conjectured to represent:
  - Information storage
  - Information transfer
    - “
  - Information modification

No quantified evidence !!

Information-theoretical preliminaries

- Shannon entropy

$$H(X) = -\sum_{x \in A_X} p(x) \log p(x)$$

- Joint entropy

$$H(X,Y) = -\sum_{xy \in A_X A_Y} p(x,y) \log p(x,y)$$

- Conditional entropy

$$H(X \mid Y) = -\sum_{xy \in A_X A_Y} p(x,y) \log p(x \mid y)$$

- Mutual information

$$I(X;Y) = \sum_{xy \in A_X A_Y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

$$I(X;Y) = H(X) - H(X \mid Y) = H(Y) - H(Y \mid X)$$

- Conditional mutual information

$$I(X;Y \mid Z) = H(X \mid Z) - H(X \mid Y,Z) = H(Y \mid Z) - H(Y \mid X,Z)$$

Measuring Information Transfer

- Mutual information was previously the de facto candidate for measuring information transfer:

$$I(X;Y) = \sum_{x,y} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}.$$

- But MI is a symmetric, static measure of shared information.
- Transfer entropy introduced by Schreiber as an asymmetric measure of dynamic info flow:

$$T_{Y \to X} = \sum_{z_n} p(z_n) \log \frac{p(x_{n+1}|x_n^{(k)}, y_n^{(l)})}{p(x_{n+1}|x_n^{(k)})}. \quad z_n = (x_{n+1}, x_n^{(k)}, y_n^{(l)})$$

- Is a mutual information between source and destination, conditioned on the past of the destination

$$T_{Y \to X} = I(Y;X'|X) = H(X'|X) - H(X'|X,Y)$$

Deriving Local Transfer Entropy (1/2)

$$T_{Y\to X} = \sum_{z_n} p(z_n) \log \frac{p(x_{n+1}|x_n^{(k)}, y_n)}{p(x_{n+1}|x_n^{(k)})}. \quad z_n = (x_{n+1}, x_n^{(k)}, y_n)$$

1. $$p(z_n) = c(z_n)/N$$

2. $$p(z_n) = \left(\sum_{m=1}^{c(z_n)} 1 \right) /N$$

3. $$T_{Y\to X} = \frac{1}{N} \sum_{z_n} \sum_{m=1}^{c(z_n)} \log \frac{p(x_{n+1}|x_n^{(k)}, y_n)}{p(x_{n+1}|x_n^{(k)})}$$

Deriving Local Transfer Entropy (2/2)

4. $$T_{Y\to X} = \frac{1}{N}\sum_{n=1}^{N} \log \frac{p(x_{n+1}|x_n^{(k)}, y_n)}{p(x_{n+1}|x_n^{(k)})},$$

$$i.e.\ \ T_{Y\to X} = \langle t_{Y\to X}(n+1)\rangle$$

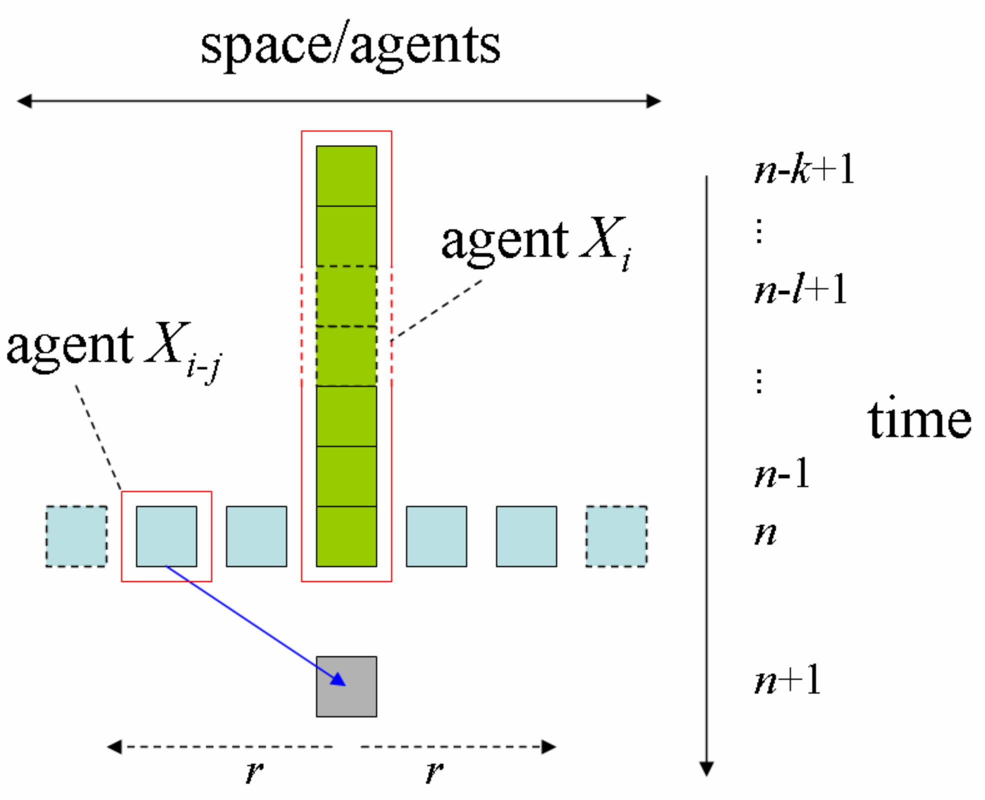
Local transfer entropy

$$t_{Y\to X}(n+1) = \log\frac{p\left(x_{n+1} \mid x_n^{(k)}, y_n\right)}{p\left(x_{n+1} \mid x_n^{(k)}\right)}$$

- Local transfer entropy is not constrained on the interval 0 < t < log *b*, unlike its averaged value:
  - t > log *b* : source is strongly informative about destination.
  - t < 0 : source is misinformative about destination, in the context of the destination's past (outcome was relatively unlikely).

Local Transfer Entropy in ordered spatiotemporal systems

$$t(i, j, n+1, k) = \log \frac{p(x_{i,n+1} | x_{i,n}^{(k)}, x_{i-j,n})}{p(x_{i,n+1} | x_{i,n}^{(k)})}$$

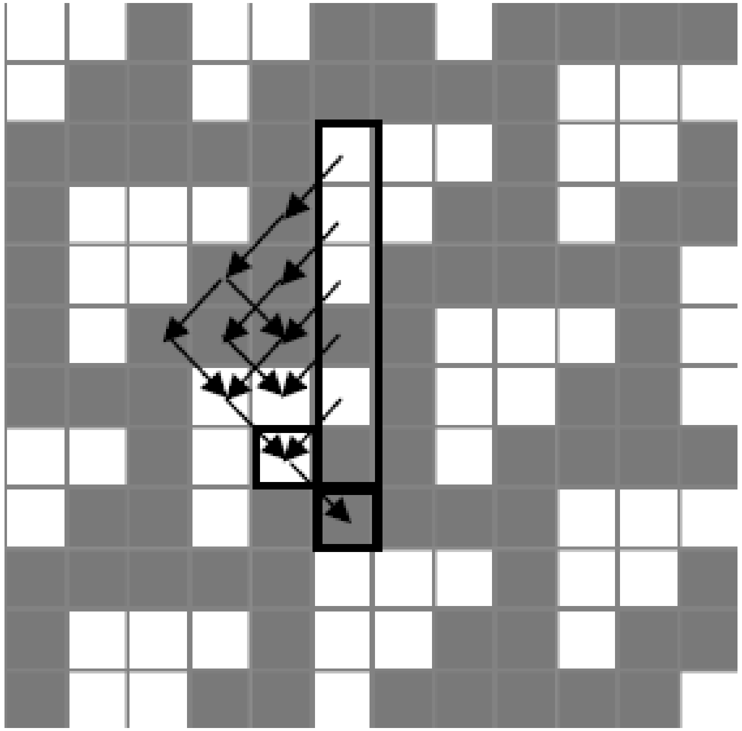


Subtlety: Accurate only in limit $k \to \infty$

- Destination's own historical values can also influence it's future states.
- This is a non-travelling form of information, like a standing wave, and is eliminated from the measurement by conditioning on the history $x_{i,n}^{(k)}$
- The most correct form is thus in the limit $k \to \infty$:

$$t(i,j,n+1) = \lim_{k\to\infty} \log \frac{p(x_{i,n+1} | x_{i,n}^{(k)}, x_{i-j,n})}{p(x_{i,n+1} | x_{i,n}^{(k)})}$$

- Use notation *t(i,j,n+1,k)* for finite k estimates.

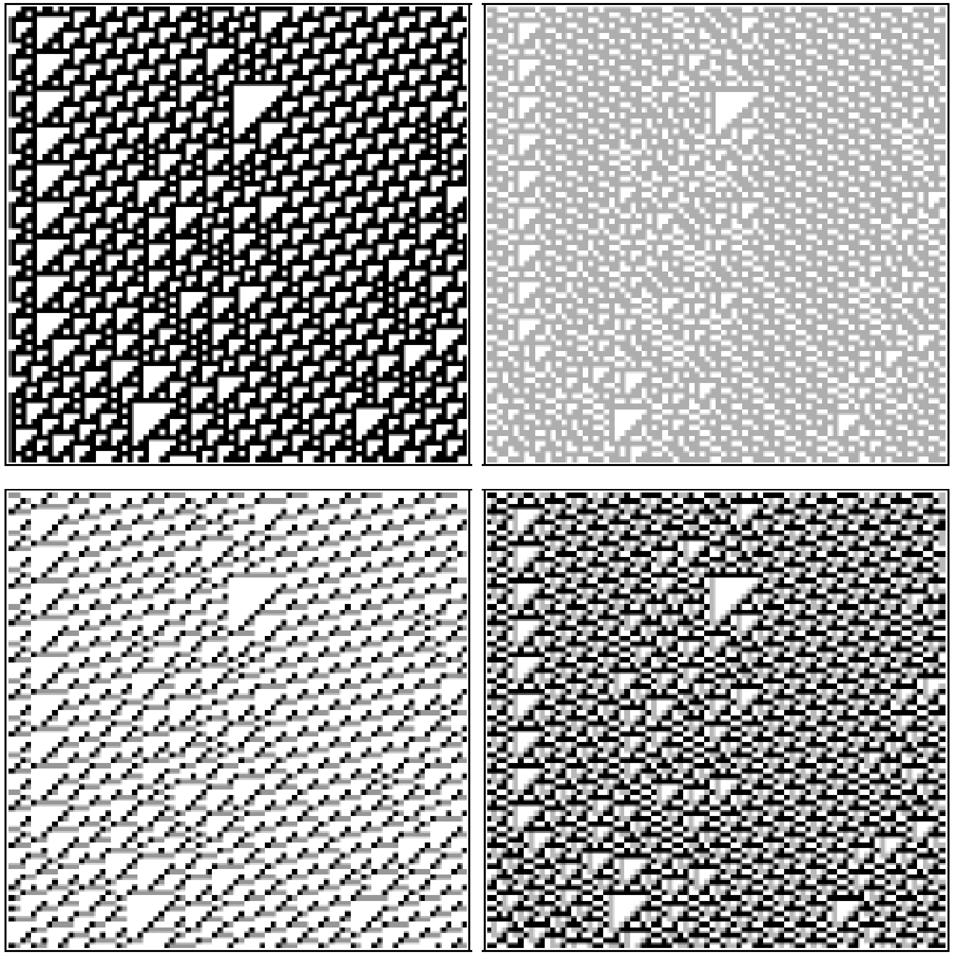


Application to CAs

- Using Elementary CAs (ECAs)
- 8 000 cells, periodic boundary conditions used.
- First 30 time steps eliminated to allow CA to settle.
- Next 500 time steps kept for estimate of probability distribution functions for $t(i,j,n,k)$
- Local TE measured at each space-time point (i,n), for each relevant channel or direction j.
- All results confirmed by several CA runs.

Base cases: k = 0,1 for ECA rule 110

Raw CA



t (i,j=1,n,k=0)

Local MI to right

t(i,j=1,n,k=1)

to right

t(i,j=-1,n,k=1)

to left

- None of these base cases distinguish the structure with any more clarity than the raw CA

Use of larger k for rule 110

Raw CA

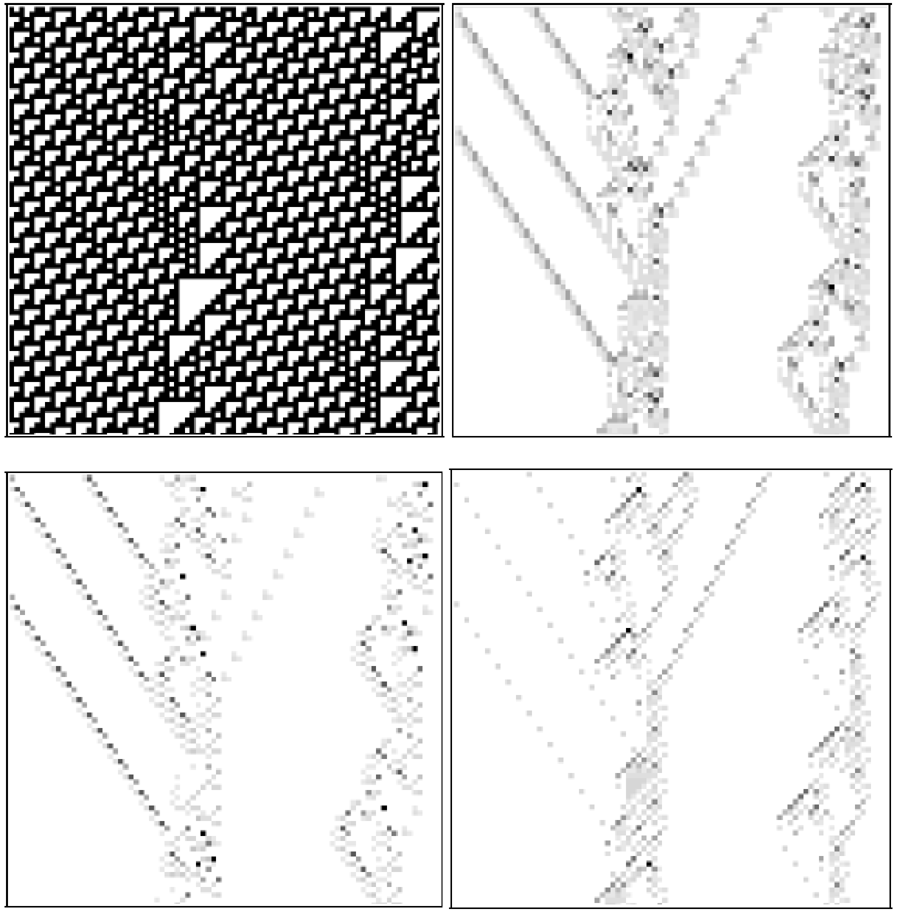
$t_s(i,n,k=6)$

t(i,j=1,n,k=6)

to right

t(i,j=-1,n,k=6)

to left

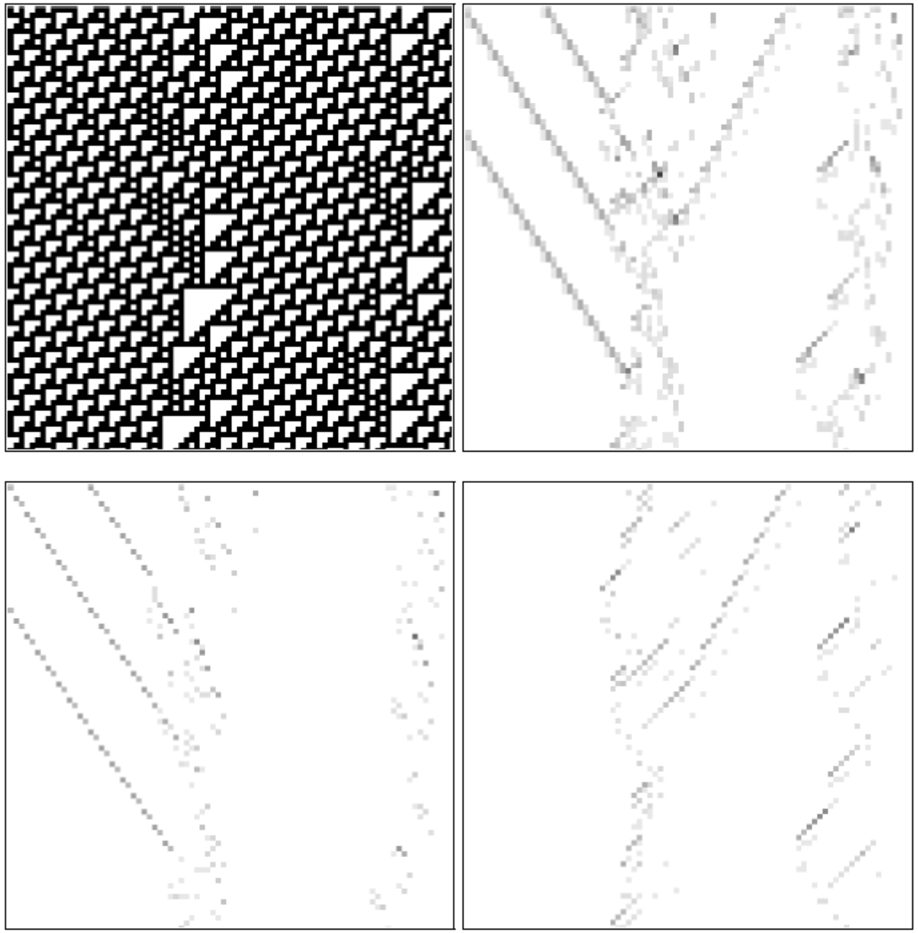


- Gliders distinguished from domain only with a minimum k.
  - Similar highlighting to existing filtering work.
  - Direction of transfer allows a more detailed quantitative view.

Approximate k → ∞ for rule 110

Raw CA

$t_s(i,n,k=16)$



$t(i,j=1,n,k=16)$

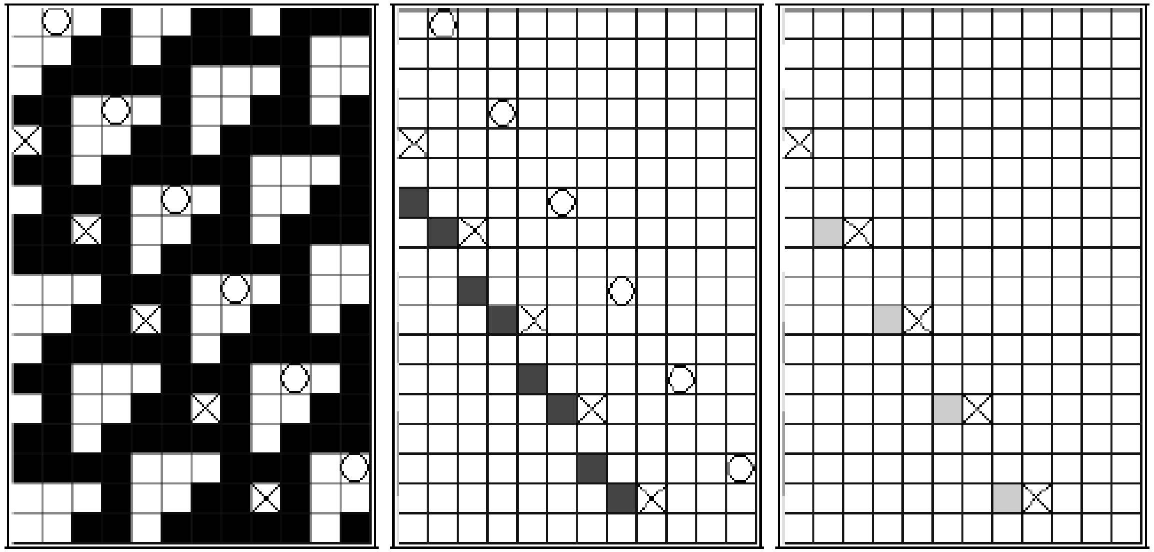
to right

$t(i,j=-1,n,k=16)$

to left

- Gliders established as dominant information transfer agents in CAs.
  - Leading edges highlighted as largest components.
  - Non-zero information transfer measured in the domain.

Close-up example



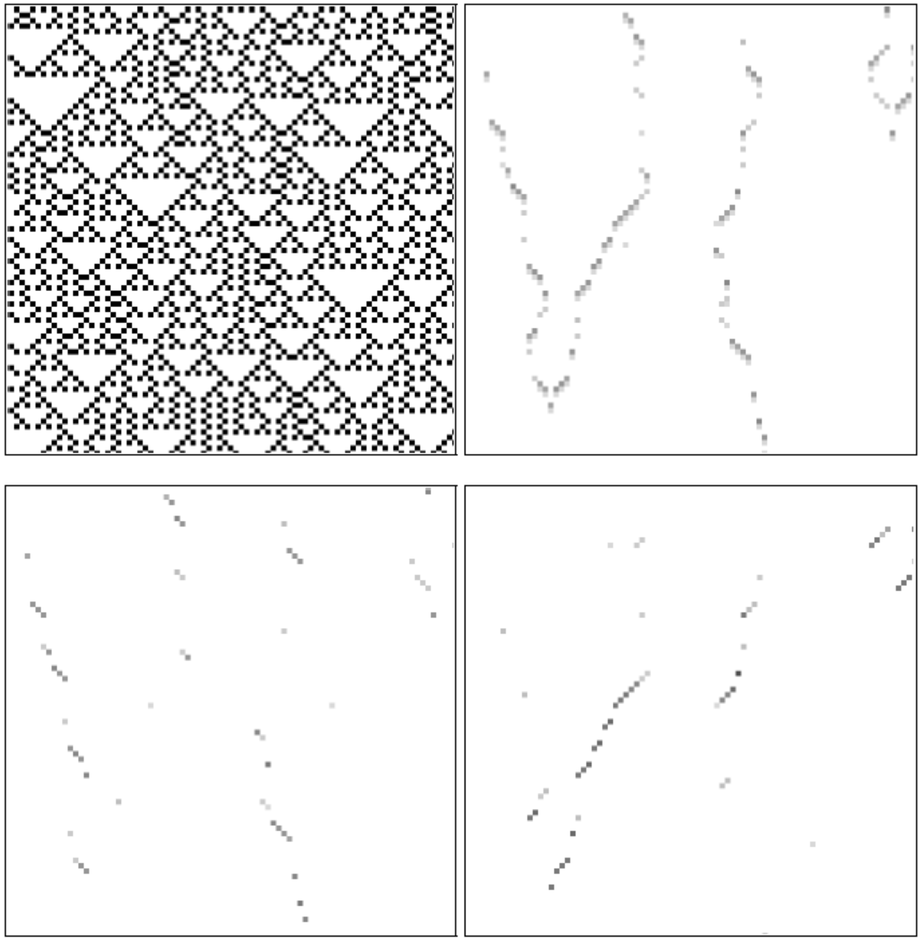
Glider in raw CA   t (i,j=1,n,k=6) > 0   t (i,j=-1,n,k=6) < 0

- Consider what the source tells us about the next state of the destination, in the context of the destination's past:
  - High transfer entropy for gliders: the source (in the glider) indicates that the periodic domain in the past of the destination will not continue.
  - Negative transfer entropy (misleading information) found on gliders for measurements orthogonal to glider motion: the source (as part of domain) is misinformative that the domain in the past of the destination will continue.

Local TE profile for rule 146

Raw CA

$t_s(i,n,k=16)$



$t(i,j=1,n,k=16)$

to right

$t(i,j=-1,n,k=16)$

to left

- Domain walls also dominant information transfer agents
  - Efficient highlighting compared to other techniques

Conclusions

- Local transfer entropy gives insights not obtainable with averaged measure:
  - Local information transfer profiles
  - Importance of parameter settings
- New filter for spatiotemporal structure and local information dynamics
- Provided evidence for long-held conjecture that particles are the dominant information transfer elements in CAs
  - Evidence that transfer entropy is appropriate info transfer measure
- Future work:
  - Complete framework for local information dynamics of distributed computation
  - Comparison to a local version of information flow
  - Apply to other complex systems and conjectures about signalling therein: e.g. microtubules, flocking/swarming.
  - Use for information driven evolution.

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Thank you