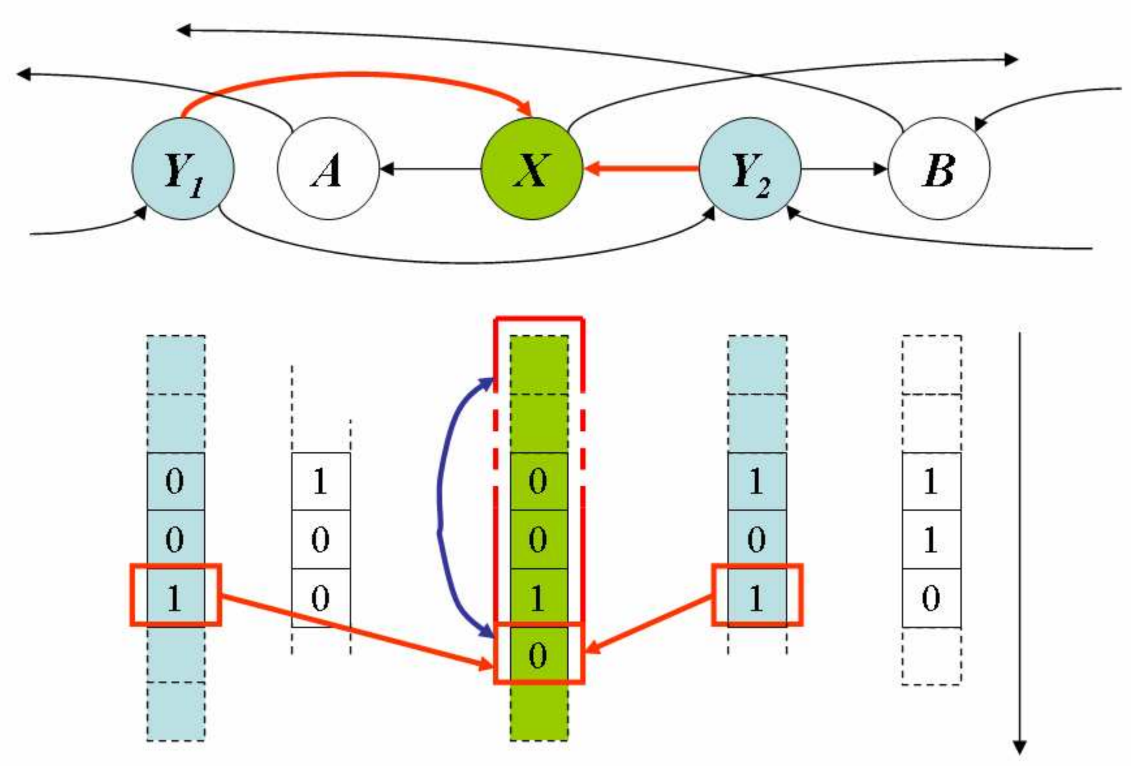

The Information Dynamics of Phase Transitions in Random Boolean Networks

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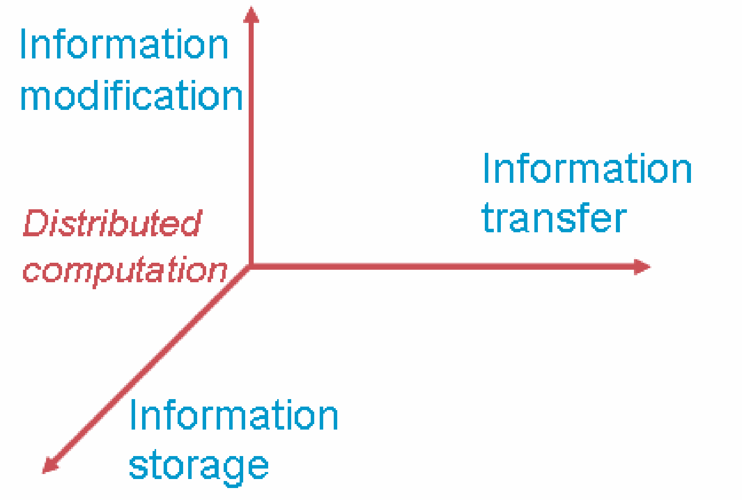
Info Dynamics of Phase Transitions in RBNs

- Aim: To study the phase transition in ordered-critical-chaotic behaviour in RBNs (as models of GRNs) from the perspective of distributed computation.



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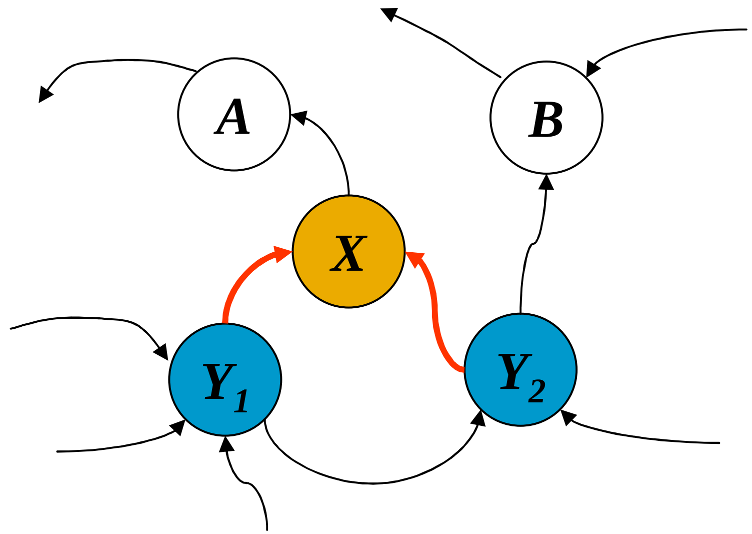


- Results: Identified maximisations in information storage and coherent information transfer on either side of the critical point.

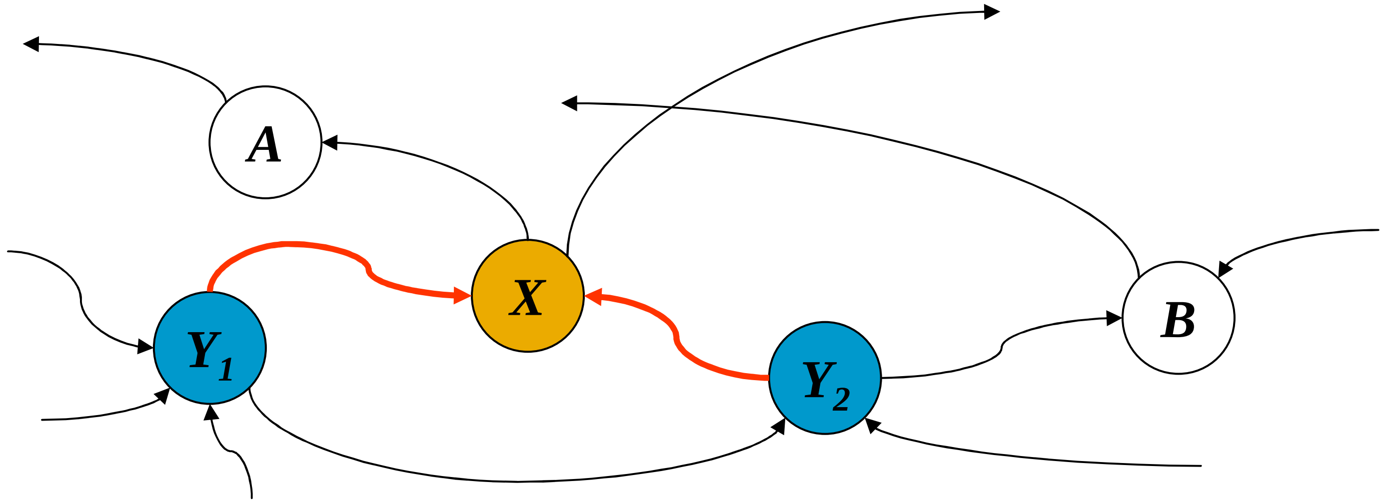
Contents

- Phase transitions in RBNs
- Information dynamics of distributed computation
- Measurement of information dynamics in RBNs
- Results and their implications

Computation in Networks: motivation

- Several authors suggest phase transitions in propagation and processing of information in networks between ordered and chaotic regimes, e.g.:
  - Message generation rate and mutual info in state of nodes in a model of computer networks (Solé and Valverde, 2001)
  - Branching ratio in a network of excitable elements (Kinouchi and Copelli, 2006)
  - Mutual info between node pairs in RBNs (Ribiero et. al., 2008)
  - Entropy in avalanche size in RBNs (Rämö et. al., 2008)
- We are particularly interested in RBNs:
  - Generality as network models with large sample space
  - Well-known phase transition
  - Popularity as models of Gene Regulatory Networks

Random Boolean Networks



RBNs used here have:

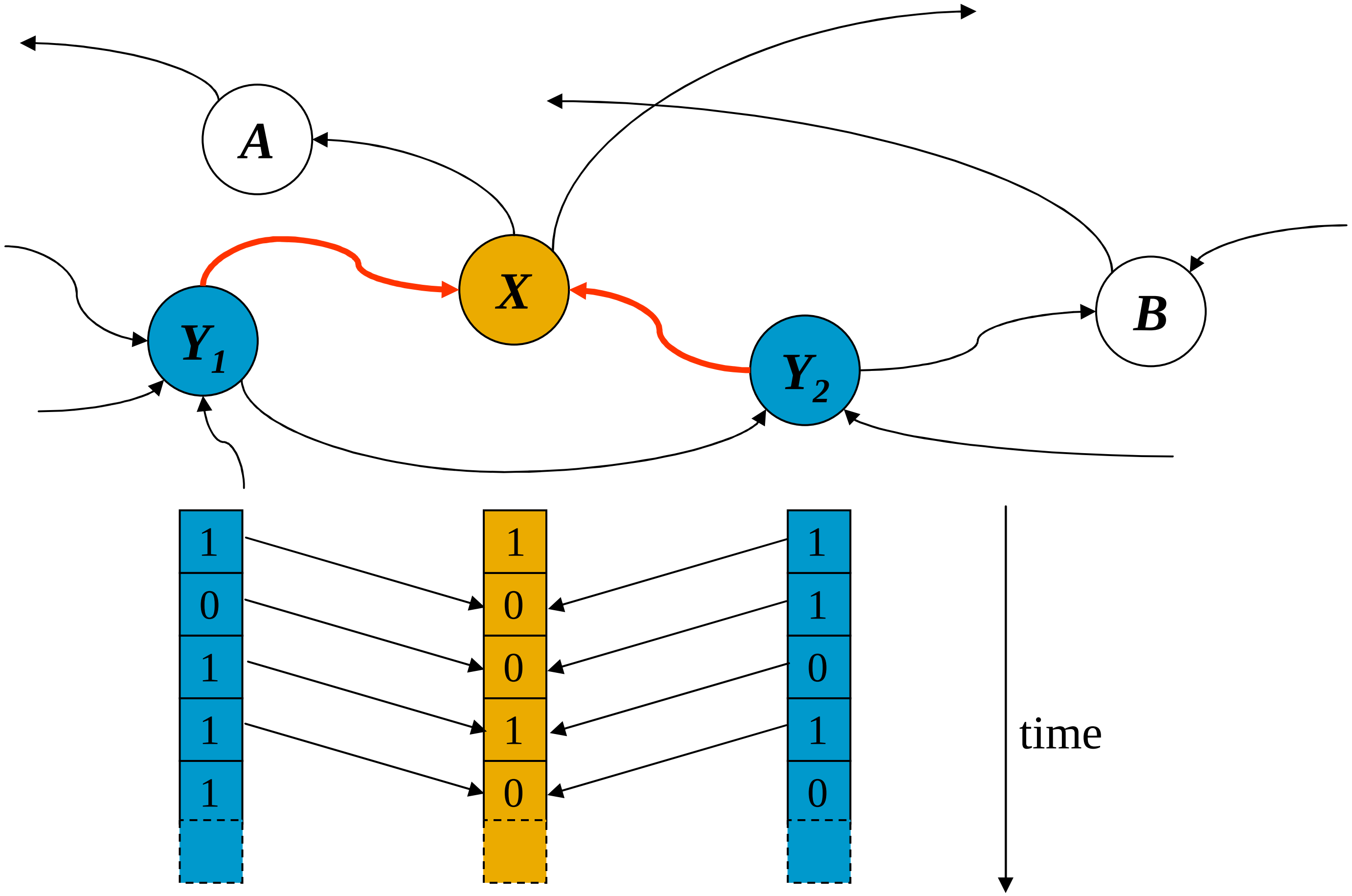
- N nodes in a directed structure,
- which is determined at random from an average in-degree $\overline{K}$.

Each node has:

- Boolean states updated synchronously in discrete time.
- Update table determined at random.

- See Kauffman "Origins of order" (1993) or Gershenson (2004).

Random Boolean Networks



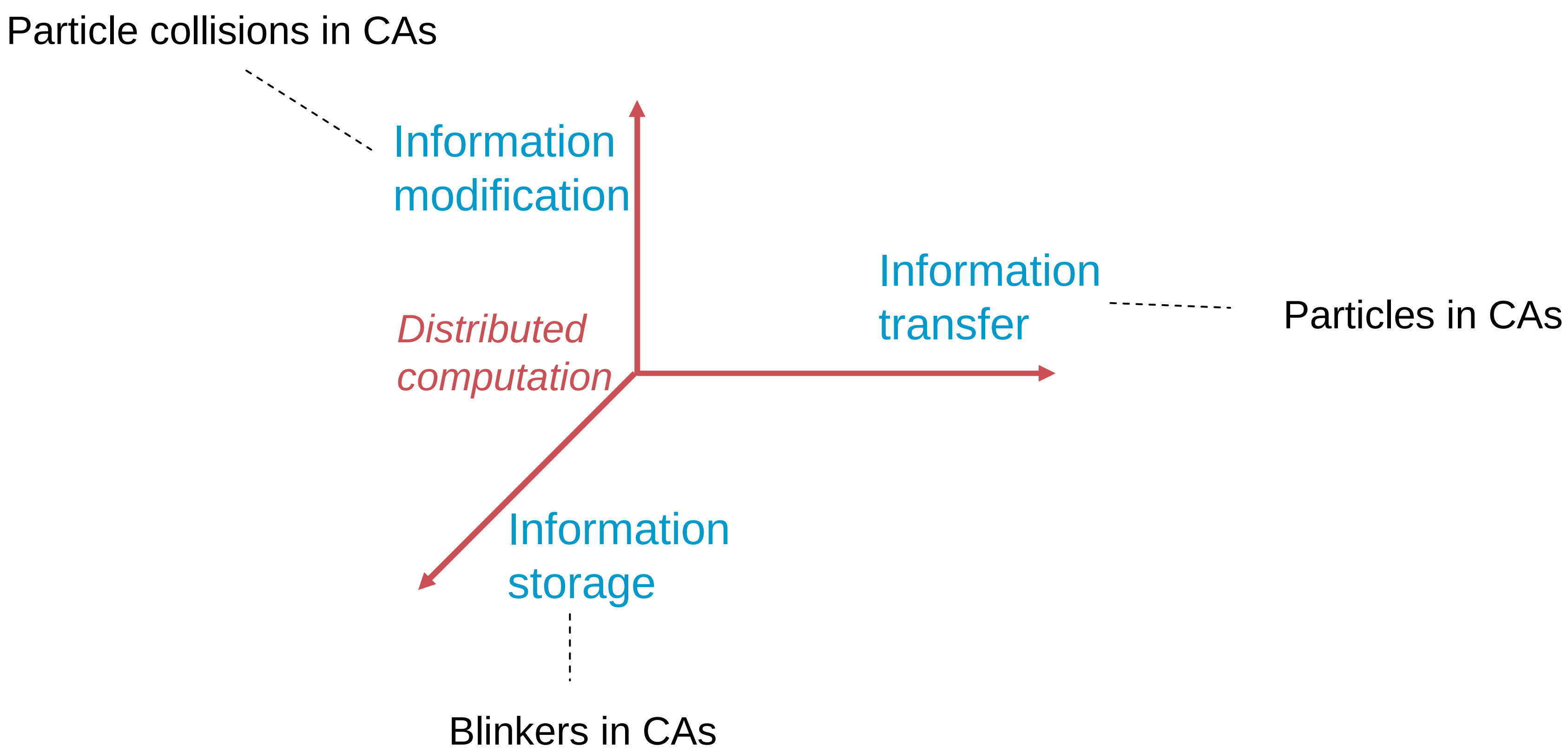
Y_1	Y_2	X
0	0	1
0	1	0
1	0	0
1	1	1

Phase transitions in RBNs

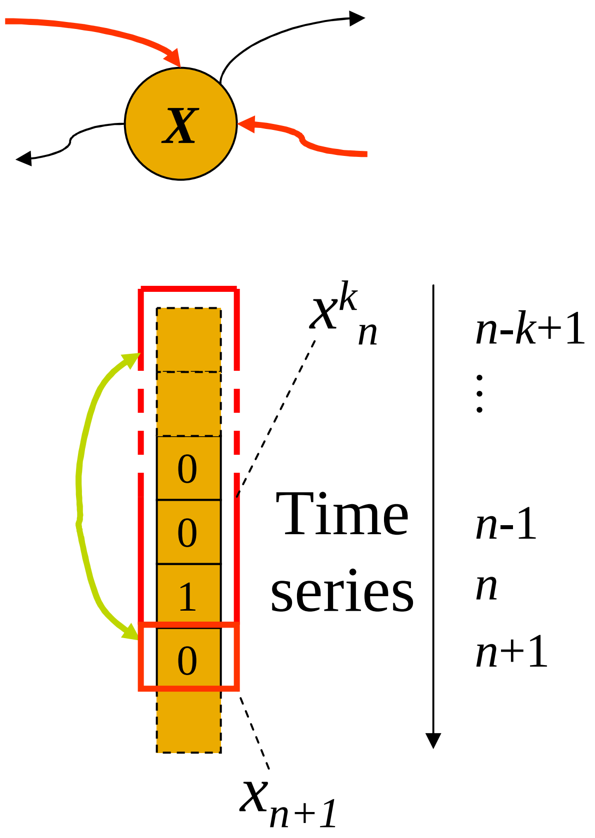
Connectivity	Low $\overline{K} < 2$	Intermediate $\overline{K} \approx 2$	High $\overline{K} > 2$
Phase	Ordered	Critical	Chaotic
Sensitivity to initial conditions	Low $\delta < 0$	Critical $\delta \approx 0$	High $\delta > 0$
Convergence of similar macro states	Strong	Uncertain	Highly divergent

Information dynamics

- Information dynamics of distributed computation in terms of 3 components of Turing universal computation:



Information storage

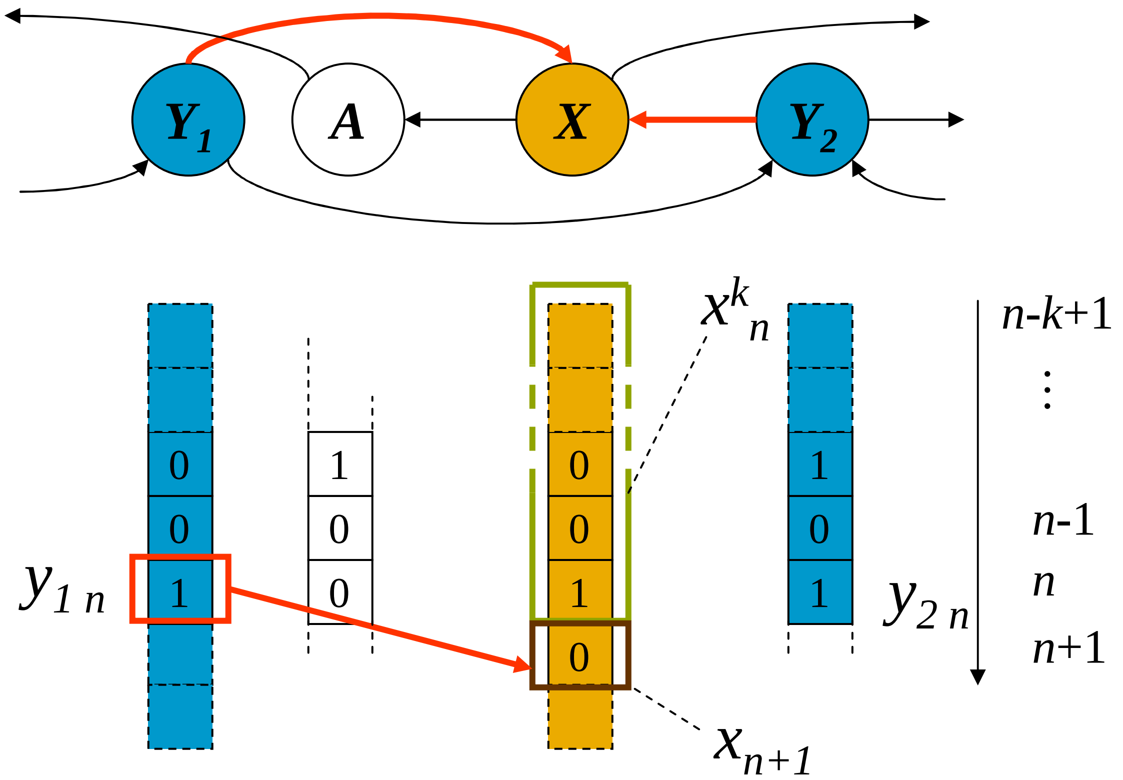


- Information storage: info in past of an agent relevant to predicting its future.
- Active info storage = mutual info between past and next step:

  $A_x(k) = I(X', X^{(k)})$

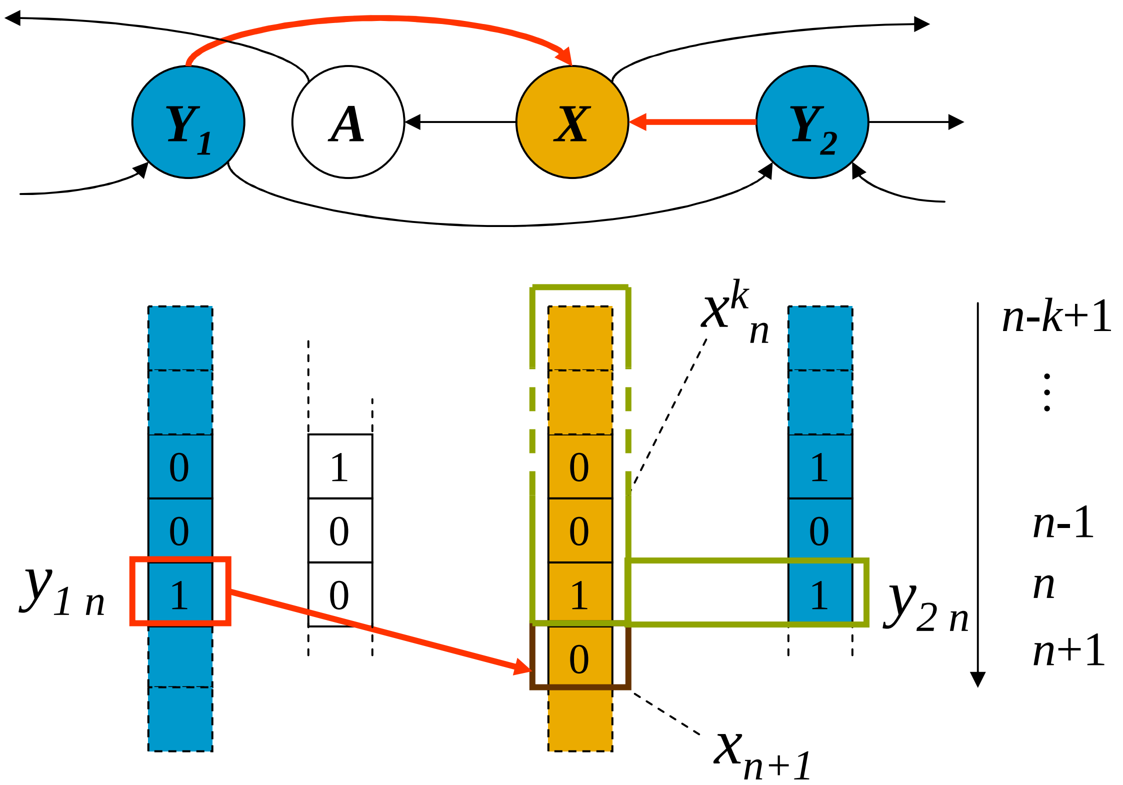
- Info to predict next state H_x =
  - Info from past $A_x(k)$ +
  - Remaining uncertainty $H_{\mu x}(k)$

Information transfer



- **Apparent** transfer entropy: mutual information between **source** and **destination** conditioned on the **past** of the destination, e.g. $T_{Y1 \rightarrow X}(k) = I(Y_1, X'; X^{(k)})$

Information transfer

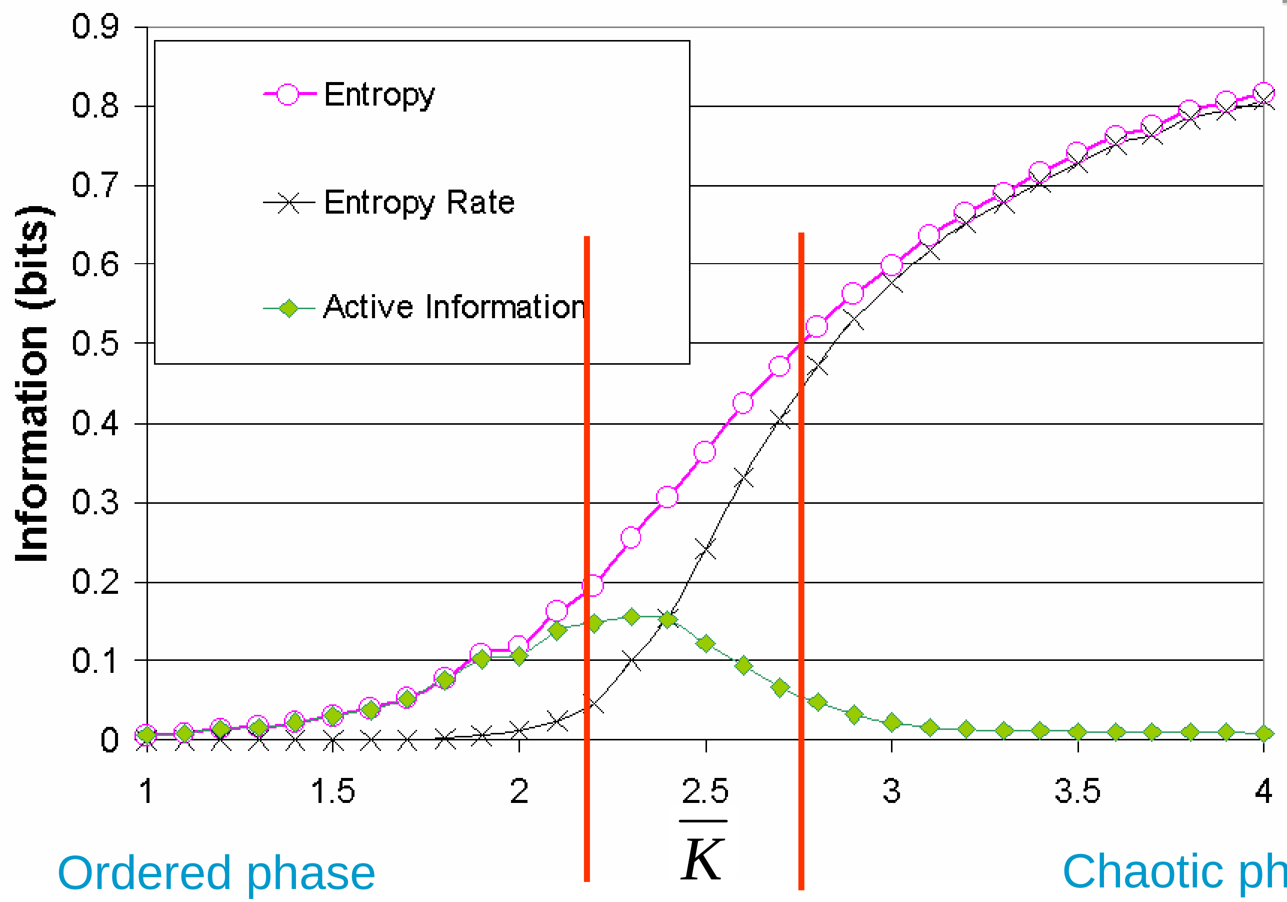


- **Complete** transfer entropy also conditions on other causal information sources, e.g. $T^C_{Y1\rightarrow X}(k)=I(Y_1,X';X^{(k)},Y_2)$

Experimental details

- **Measure the average information dynamics as a function of average connectivity $\overline{K}$.**
- Networks of 250 nodes
- Poissonian distributed in-degree for each node based on average in-degree $1 \leq \overline{K} \leq 4$
- RBNs modelled with enhancements to the RBNLab software.
- For each $\overline{K}$:
  - Average the measures over at least 250 RBNs.
  - For each RBN:
    - Average the measures over at least 50 nodes/links, using observations of 400 steps of RBN evolution from at least 4000 random initial states.
- Use $k \geq 13$ for information dynamics calculations.

Results: domination of phases

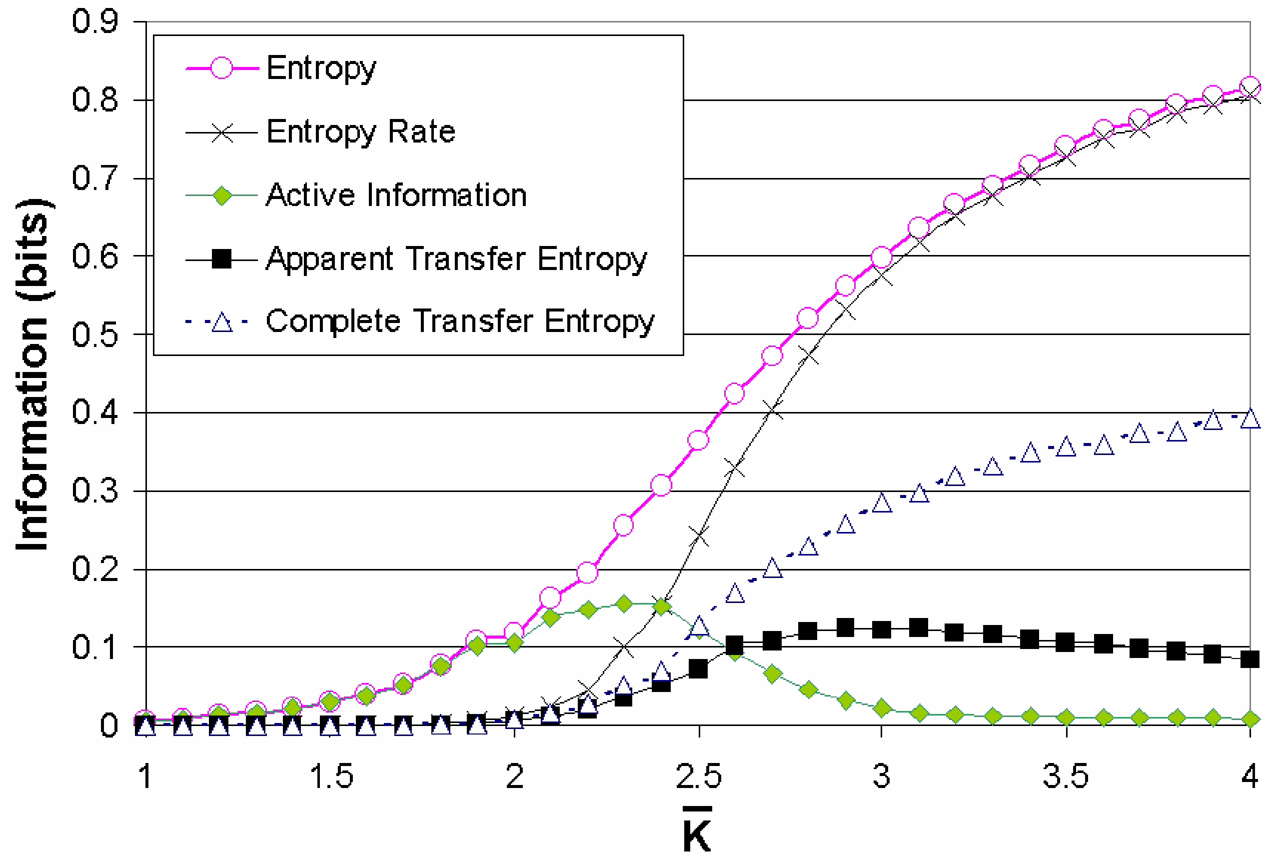


Ordered phase dominated by information storage

Balance near critical phase

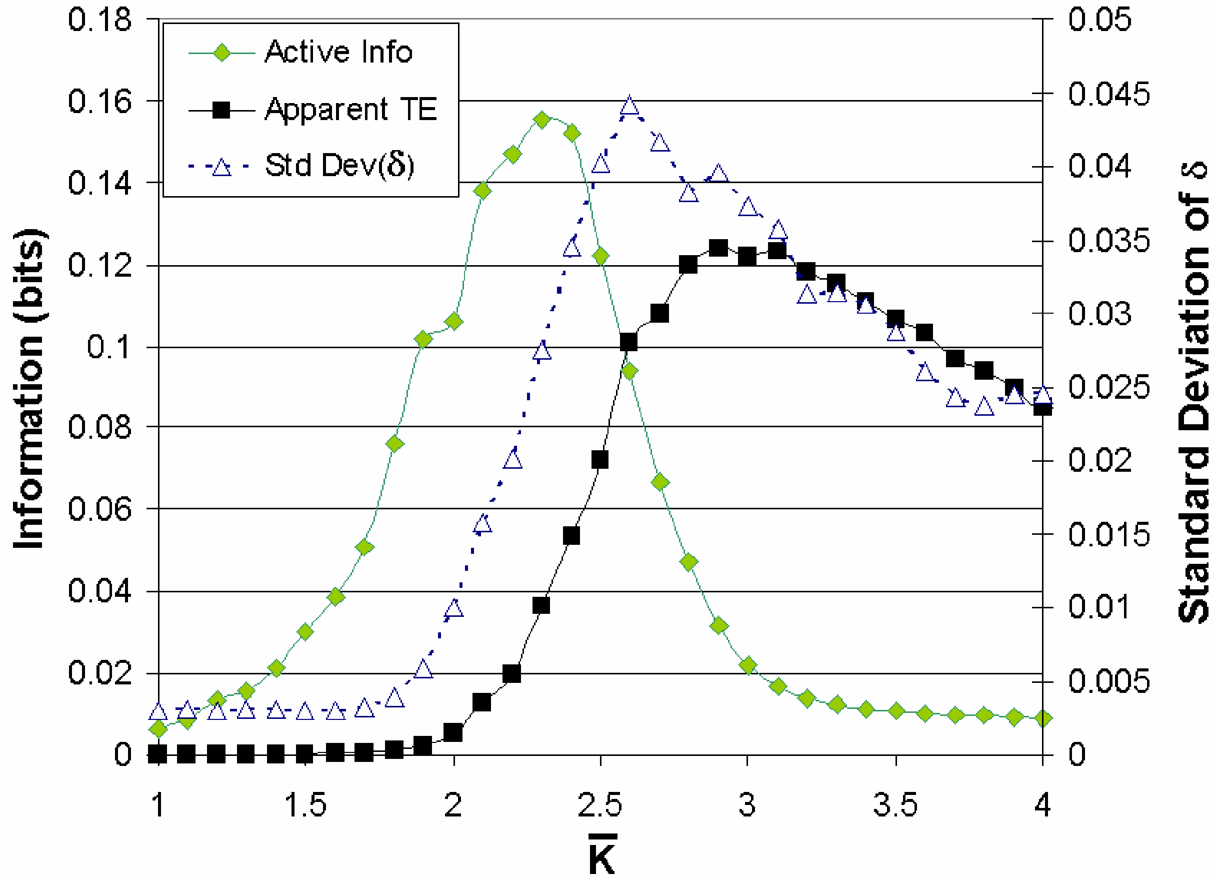
Chaotic phase dominated by information transfer

Results: make-up of information transfer



- Apparent transfer entropy peaks close to critical phase
- Complete transfer entropy continues to rise in chaotic regime

Results: positioning around critical point



- Information storage peaks slightly within the ordered regime.
- (Coherent) Information transfer peaks slightly within the chaotic regime.

Conclusions

- Quantified the fundamental nature of distributed computation around phase transitions in RBNs.
- First exploration of info dynamics of an order-chaos phase transition:
  - Provides insights into maximisation versus intermediate level of information transfer near critical point.
  - Results may be pertinent to similar phase transitions.
- Future work:
  - Further explore relation of topology and information dynamics.
  - Use info dynamics to explain other measures of phase transitions.
  - Explore whether info dynamics can be used to drive evolution or self-tuning adaptation of RBNs to produce critical networks.

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