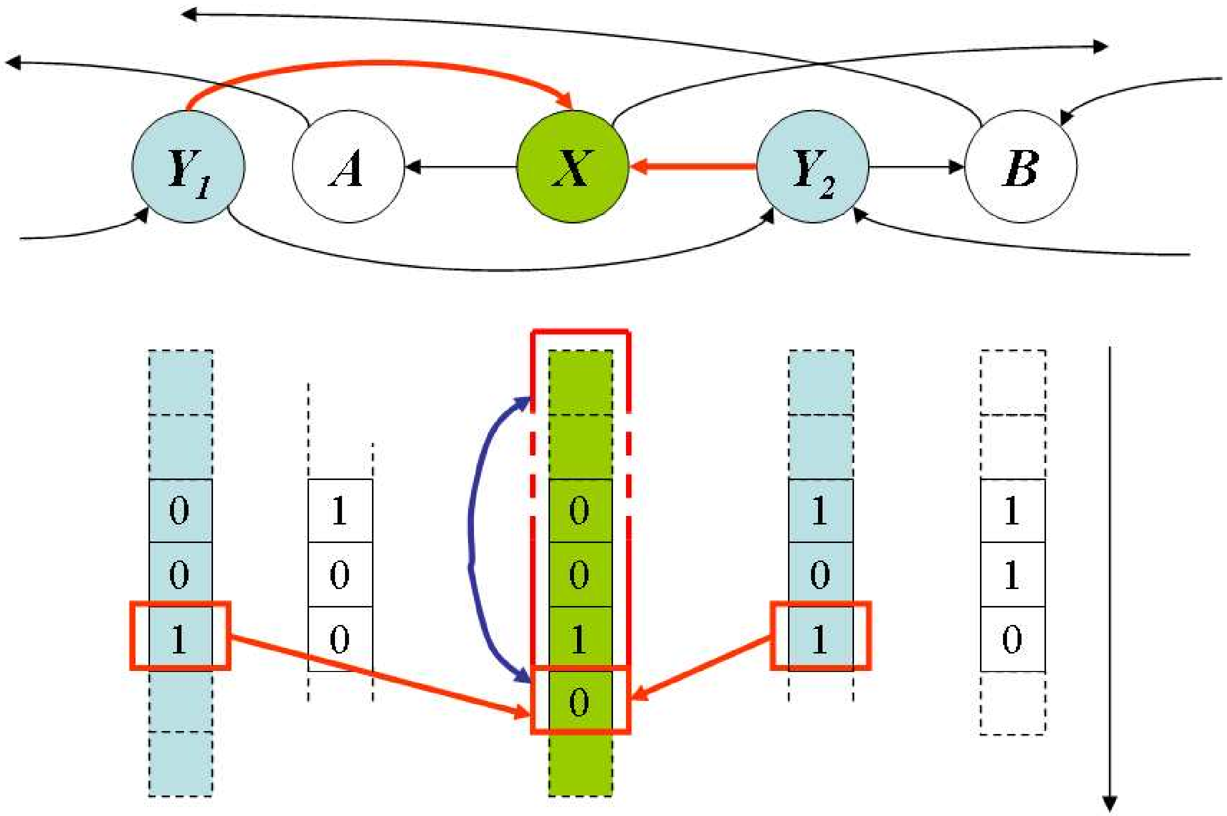

Coherent local information dynamics in complex computation

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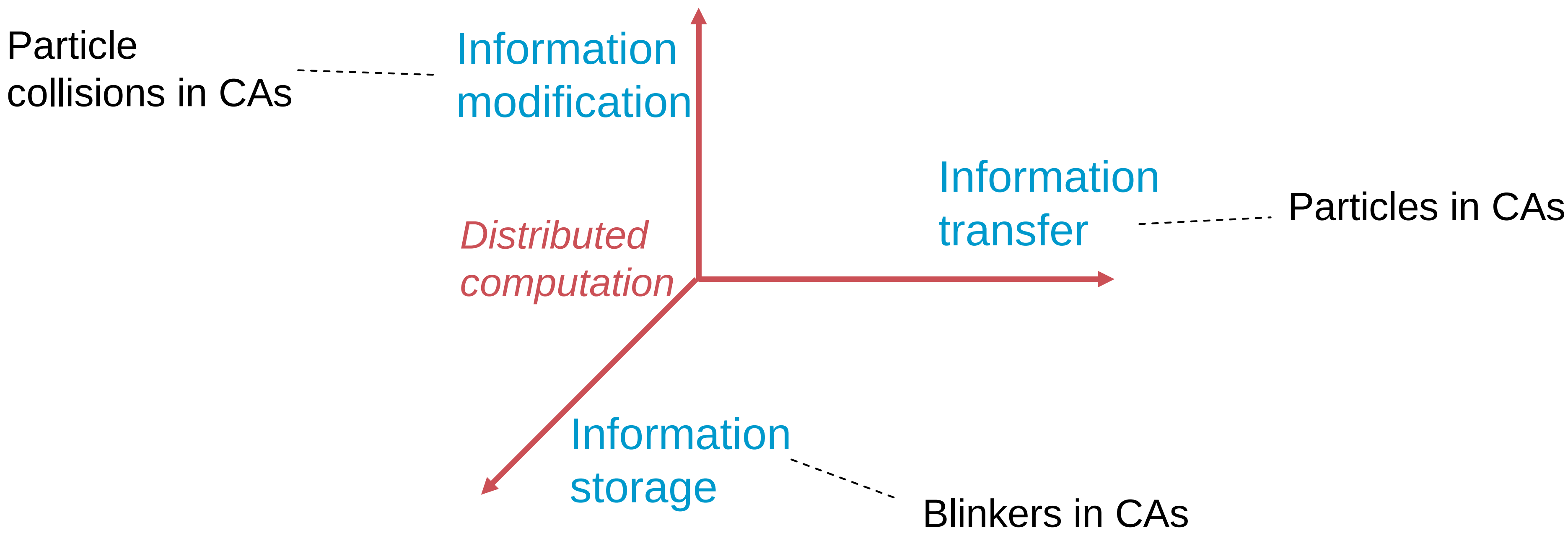
Computation: memory, signalling, processing

- We talk about computation as:
  - Memory
  - Signalling/Comms
  - Processing
- We quantify computation in terms of:
  - Information storage
  - Information transfer
  - Information modification
- Distributed computation is any process that involves these features, e.g.:
  - Time evolution of cellular automata
  - Information processing in the brain
  - Gene regulatory networks computing cell type
  - Flocks computing their collective heading
  - Ant colonies computing the most efficient routes to food sources
  - The universe is computing it's own future!
- Seth Lloyd in "Computing the universe"
  - "… all physical systems register and process information … by understanding how the universe computes, we can understand why it is complex."
- Many conjectures about distributed computation in these systems, e.g.:
  - ✓ Computation by gliders in cellular automata
  - ~ Maximisation of computational capabilities in order-chaos phase transitions

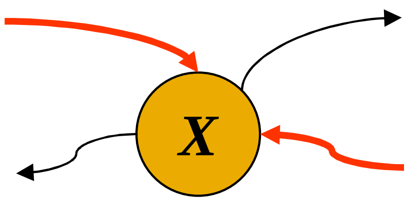
Local information dynamics

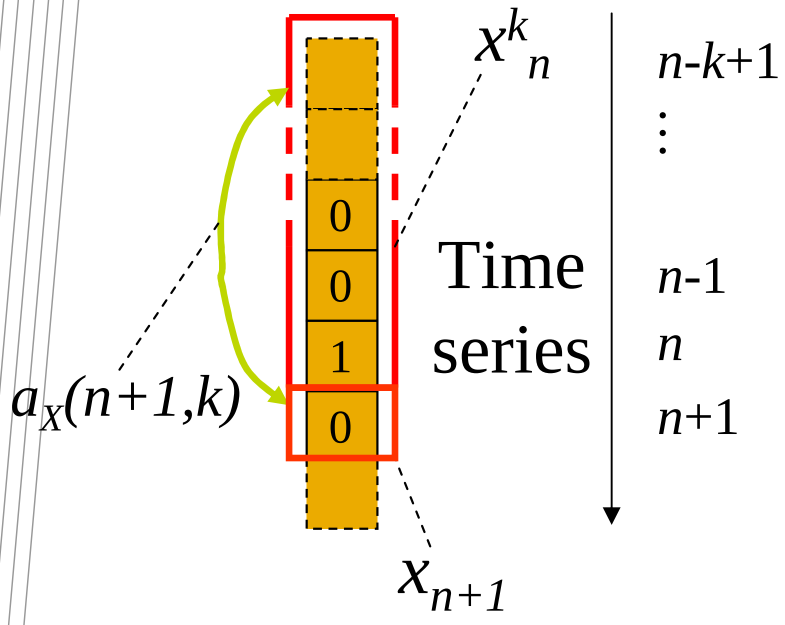
- We quantify the information dynamics of distributed computation in terms of 3 components of Turing universal computation:



- We focus on the local scale of info dynamics in space-time
  - This is typically a better characterisation of computation than averages; e.g. how much info is transferred from X_1 to X_2 at time n?

Information storage





- Information storage: info in past of an agent relevant to predicting its future.
- Active info storage = mutual info between past and next step:

$$A_X(k) = I(X; X^{(k)})$$

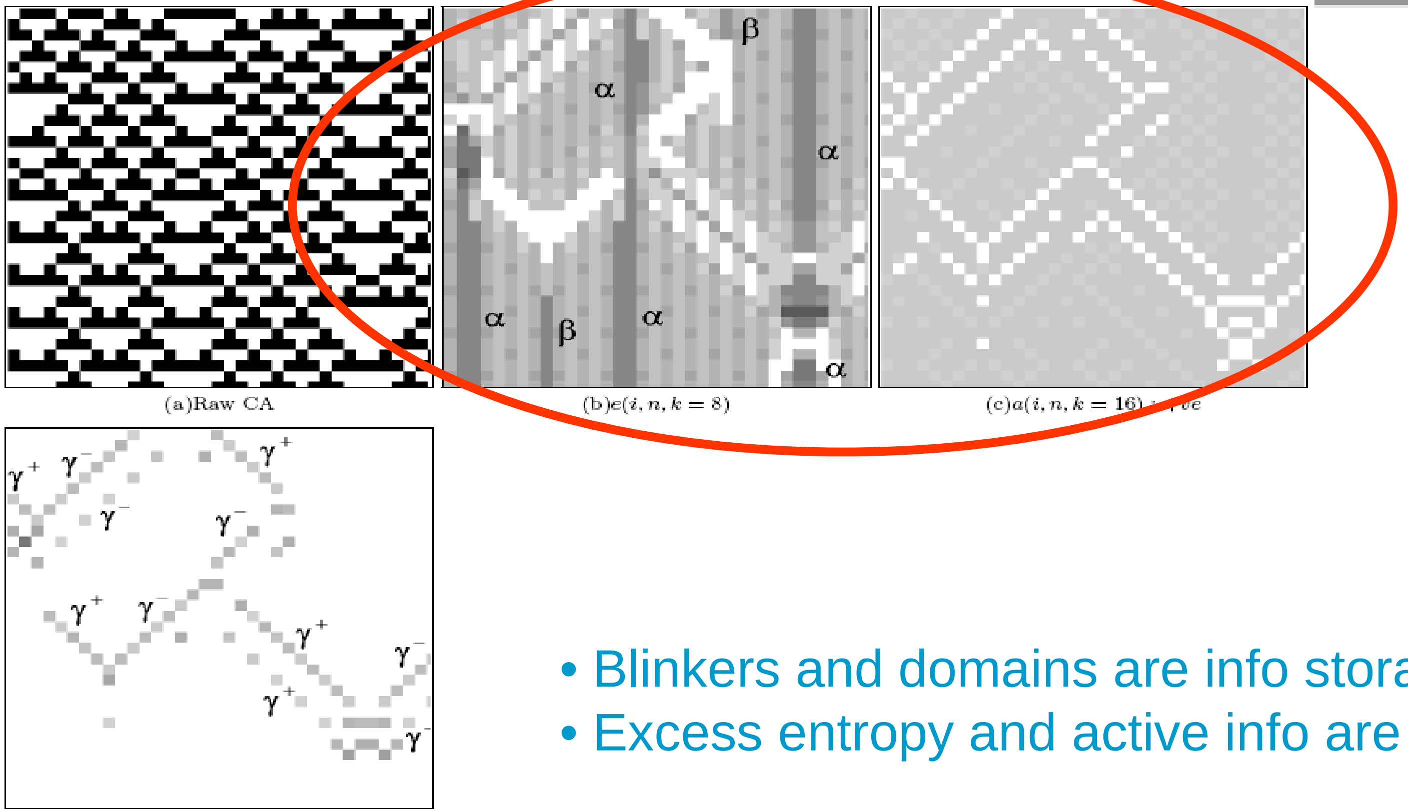
- Is the average of a *local* active information storage at each time point:

$$A_X(k) = < a_X(n,k) >_n$$

$$a_X(n,k) = \log_2 \frac{p(x_n^k, x_{n+1})}{p(x_n^k)p(x_{n+1})}$$

- Compare to excess entropy

Local information storage in CAs: rule 54



(a)Raw CA (b)$e(i, n, k = 8)$ (c)$a(i, n, k = 16)$: +ve

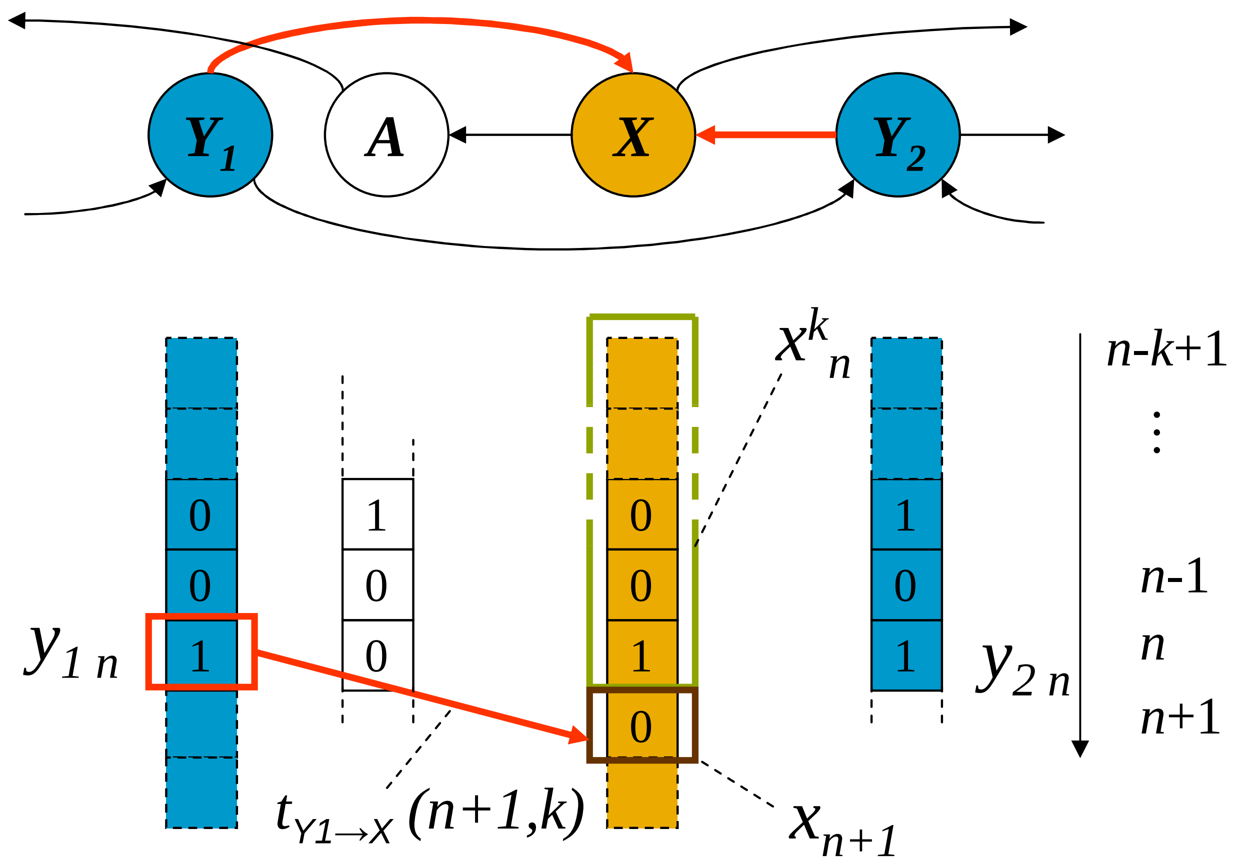
(d)$a(i, n, k = 16)$: −ve

← Information storage

- Blinkers and domains are info storage elements.
- Excess entropy and active info are complementary

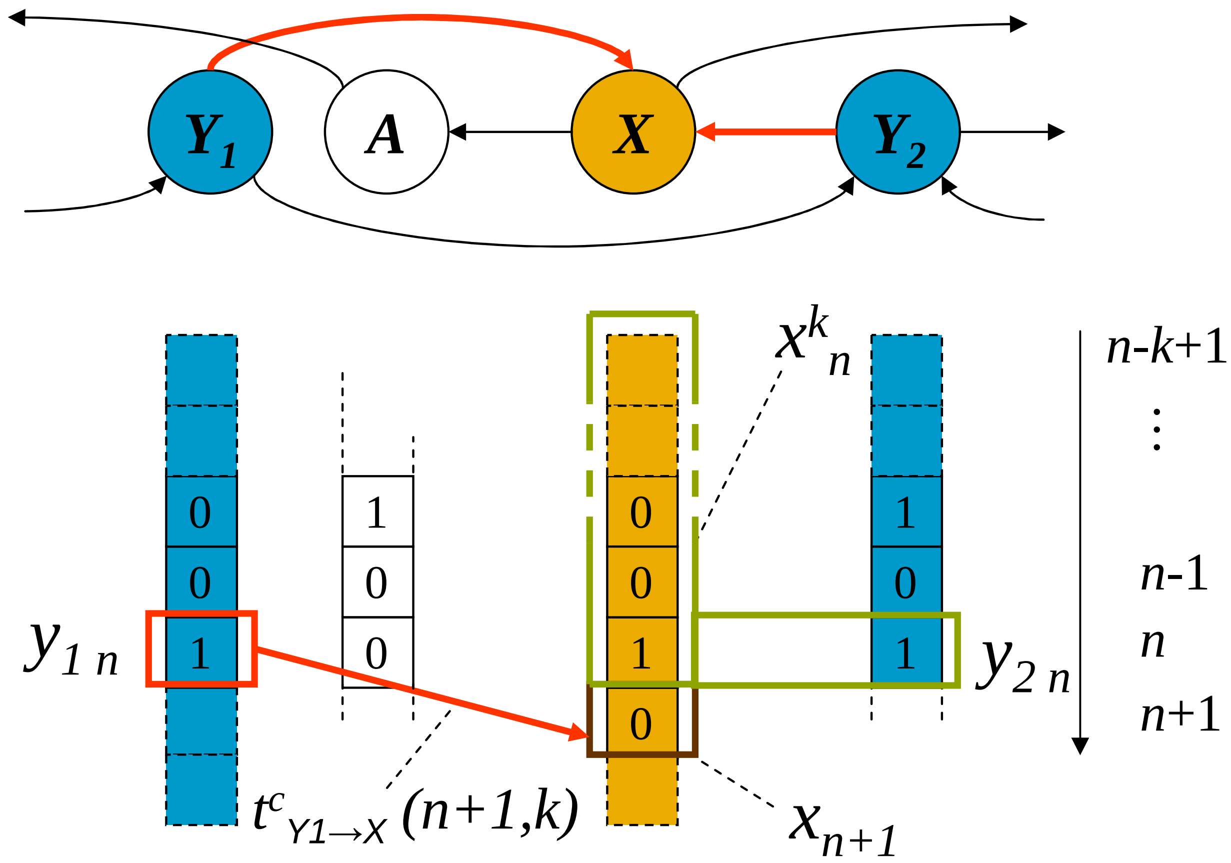
CAs

Information transfer



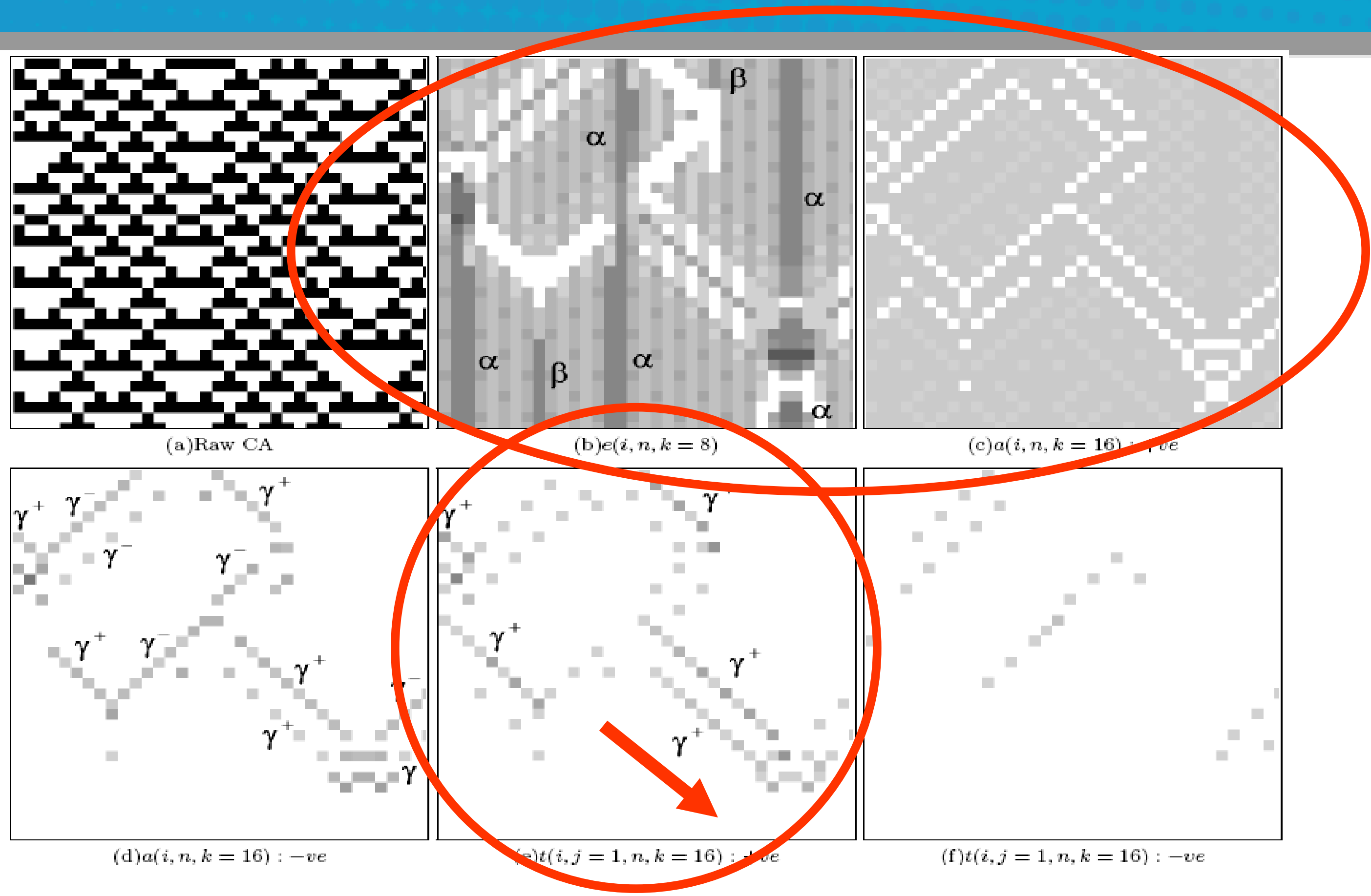
- **Apparent** transfer entropy: mutual information between **source** and **destination** conditioned on the **past** of the destination, e.g. $T_{Y1\rightarrow X}(k)=I(Y_1,X';X^{(k)})$

Information transfer



- **Complete** transfer entropy also conditions on other causal information sources, e.g. $T^C_{Y1\rightarrow X}(k)=I(Y_1,X';X^{(k)},Y_2)$

Local information transfer in CAs: rule 54

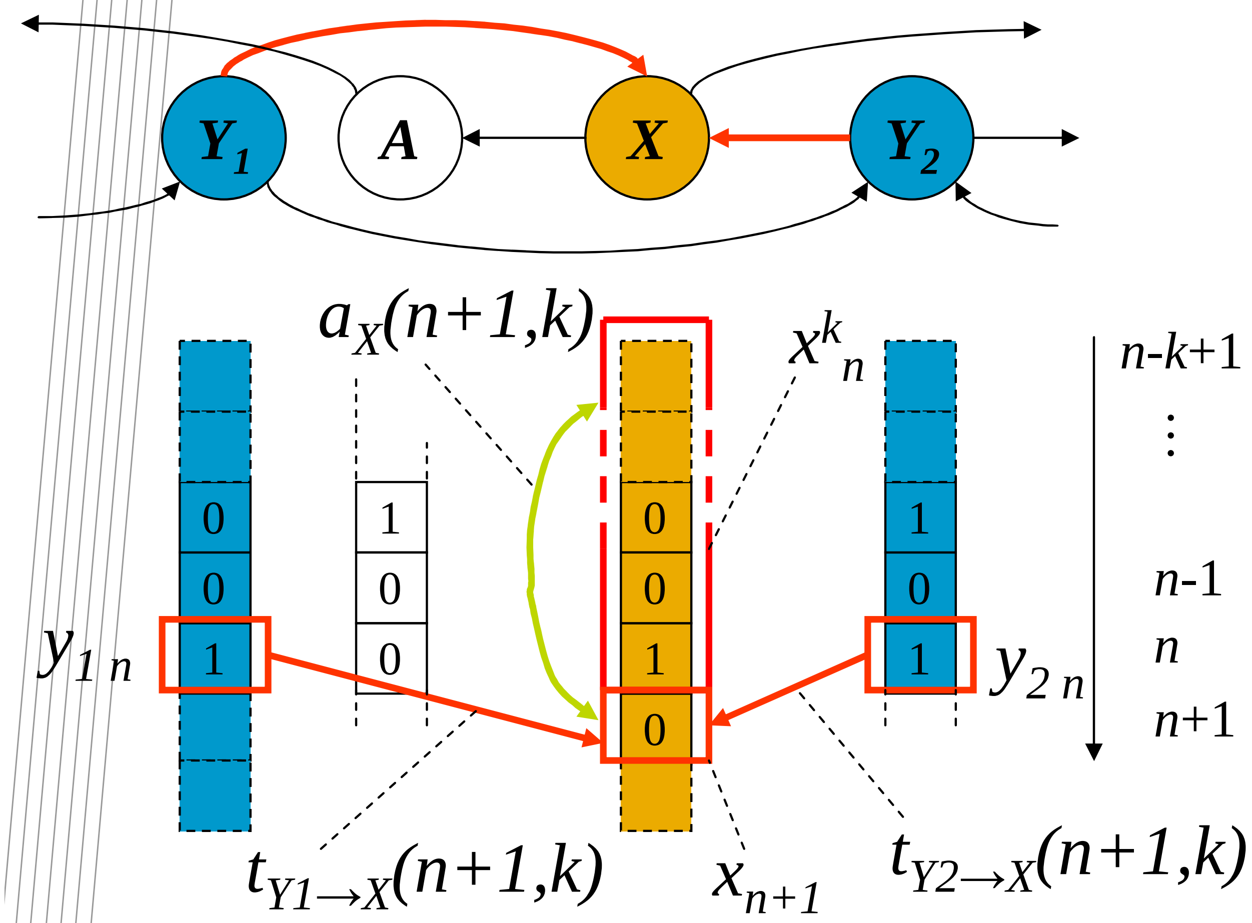


← Information storage

← Information transfer

- Gliders are info transfer agents

Information modification



- Define **Local Separable Information** as:

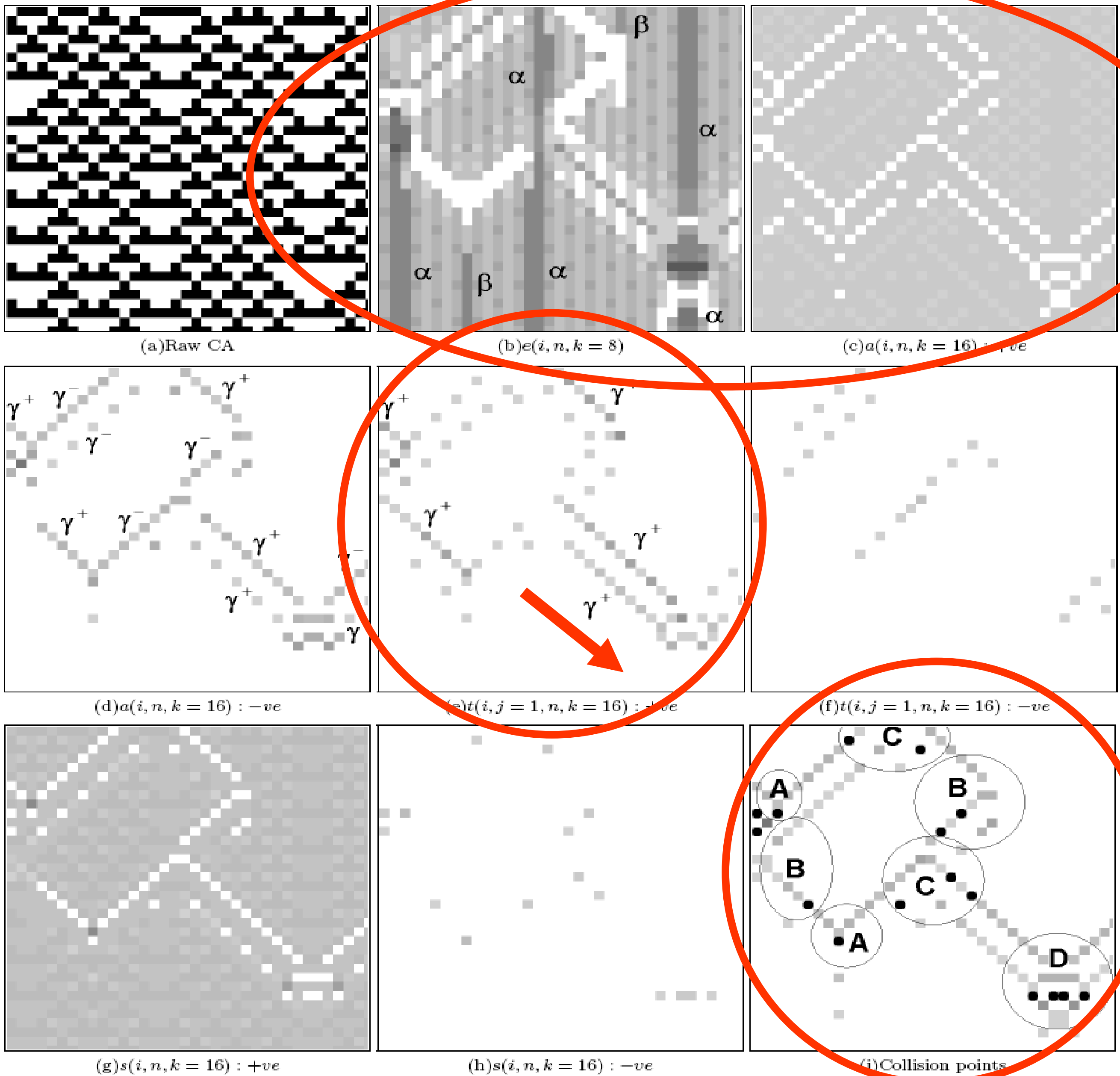
$$s_X(n) = a_X(n) + \sum_{Y \in V, Y \neq X} t_{Y \to X}(n)$$

- Average over all time steps to get $S_X(k)$.
- s > 0: trivial info modification.
- s < 0: non-trivial info modification, where sources interact.

Also comparing to a local measure of irreversibly destroyed information:

$$h(x_{i,n} | \ldots, x_{i-1,n+1}, x_{i,n+1}, x_{i+1,n+1}, \ldots)$$

Local information modification in CAs: rule 54

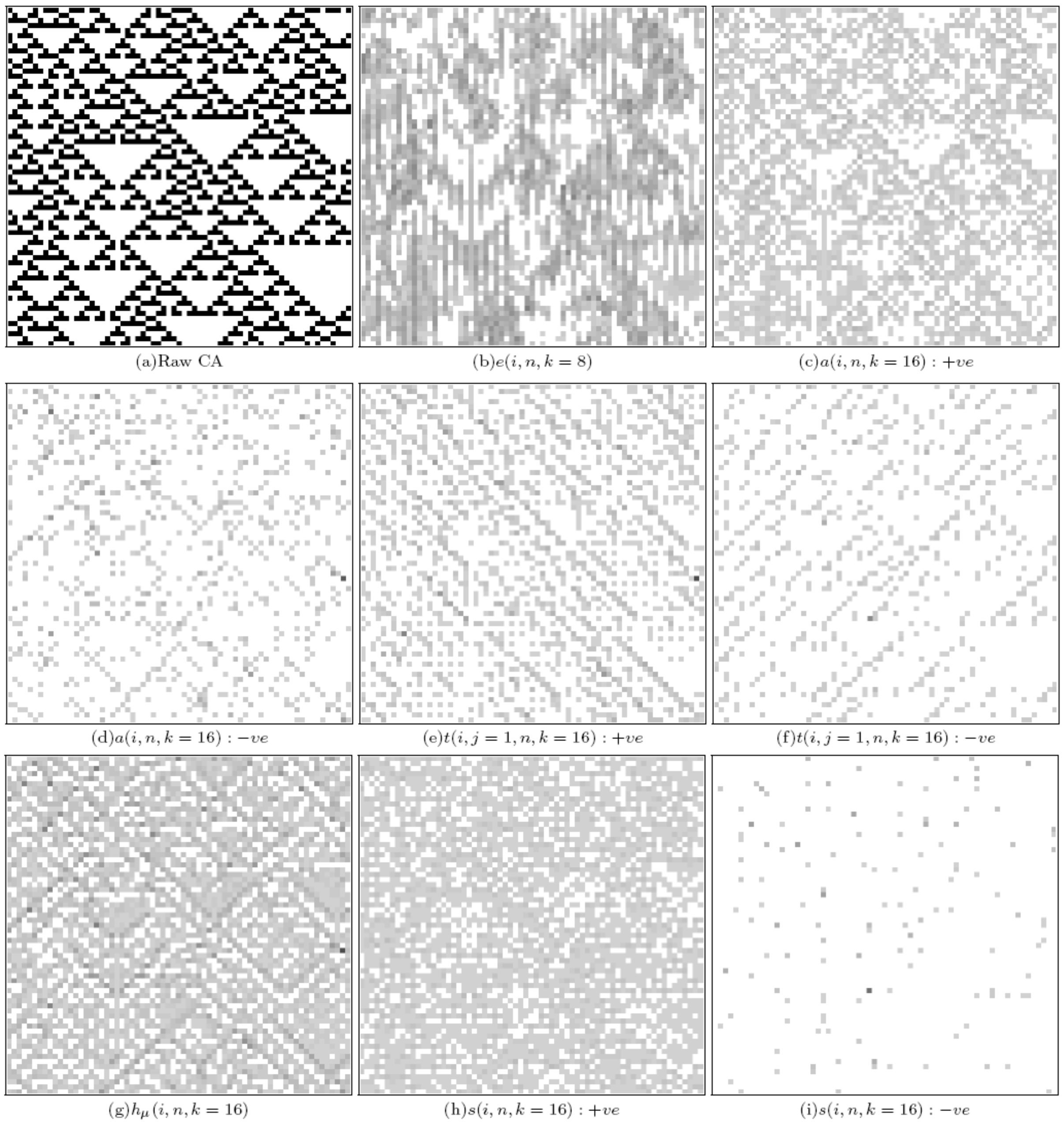


← Information storage

← Information transfer

← Information modification

Local information dynamics in CAs: rule 22

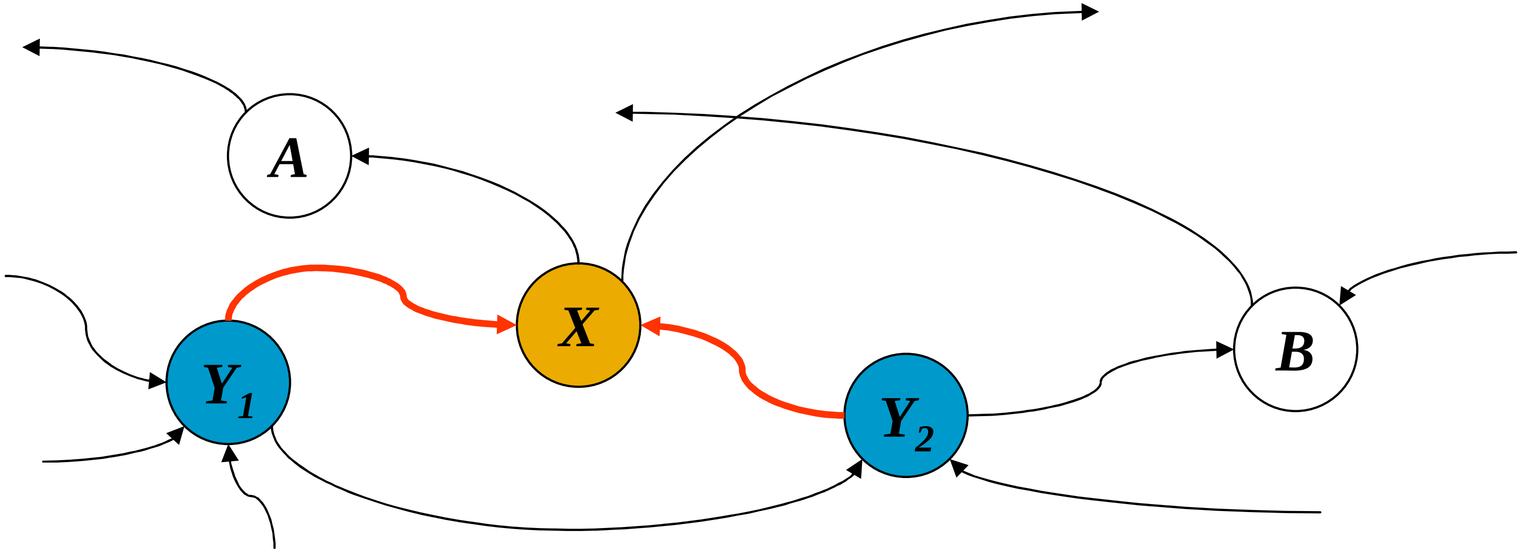


← Information storage

← Information transfer

← Information modification

Random Boolean Networks



RBNs used here have:

- N nodes in a directed structure,
- which is determined at random from an average in-degree $\overline{K}$.

Each node has:

- Boolean states updated synchronously in discrete time.
- Update table determined at random.

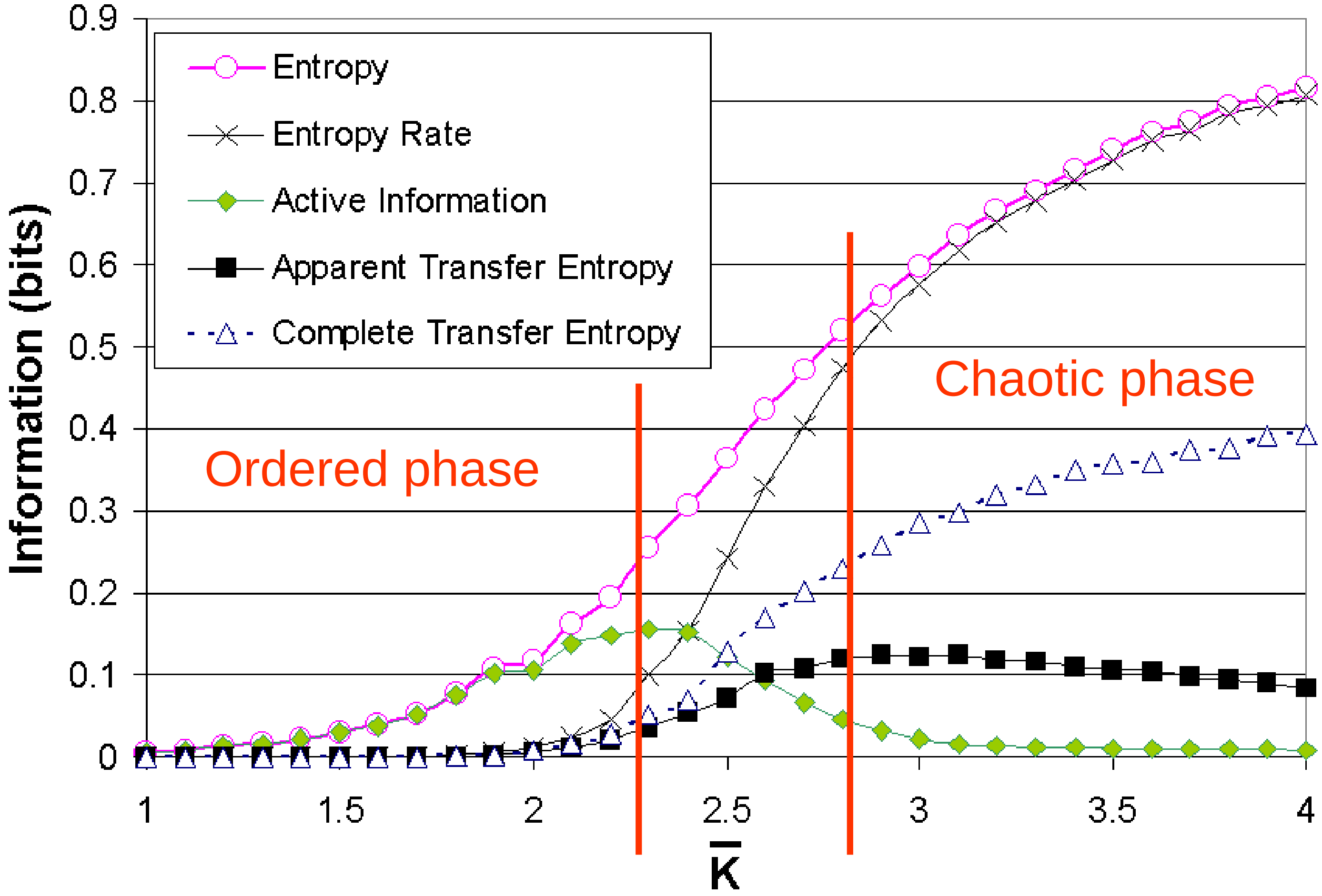
- See Kauffman "Origins of order" (1993) or Gershenson (2004).

Phase transitions in RBNs

Connectivity	Low $\overline{K} < 2$	Intermediate $\overline{K} \approx 2$	High $\overline{K} > 2$
Phase	Ordered	Critical	Chaotic
Sensitivity to initial conditions	Low $\delta < 0$	Critical $\delta \approx 0$	High $\delta > 0$
Convergence of similar macro states	Strong	Uncertain	Highly divergent

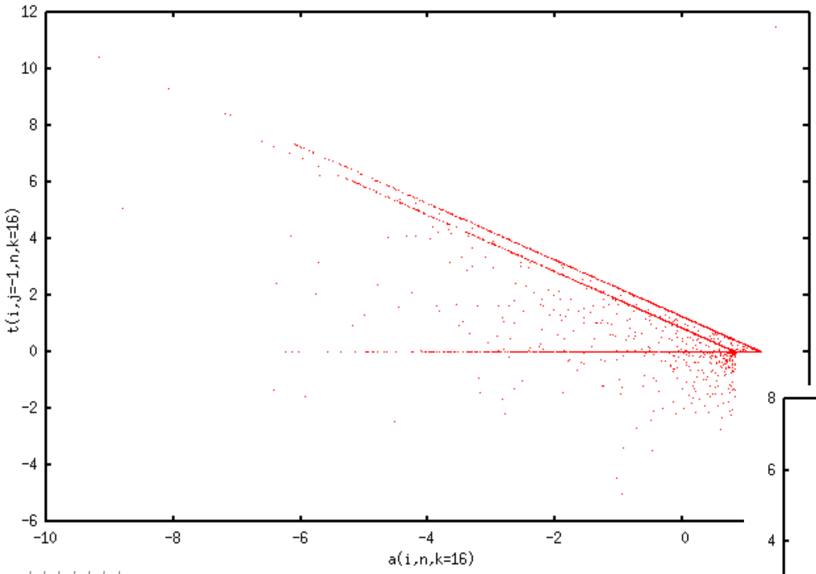
- Much diversity in behaviour of individual nodes and sampled networks

Average info dynamics through phase transition in RBNs

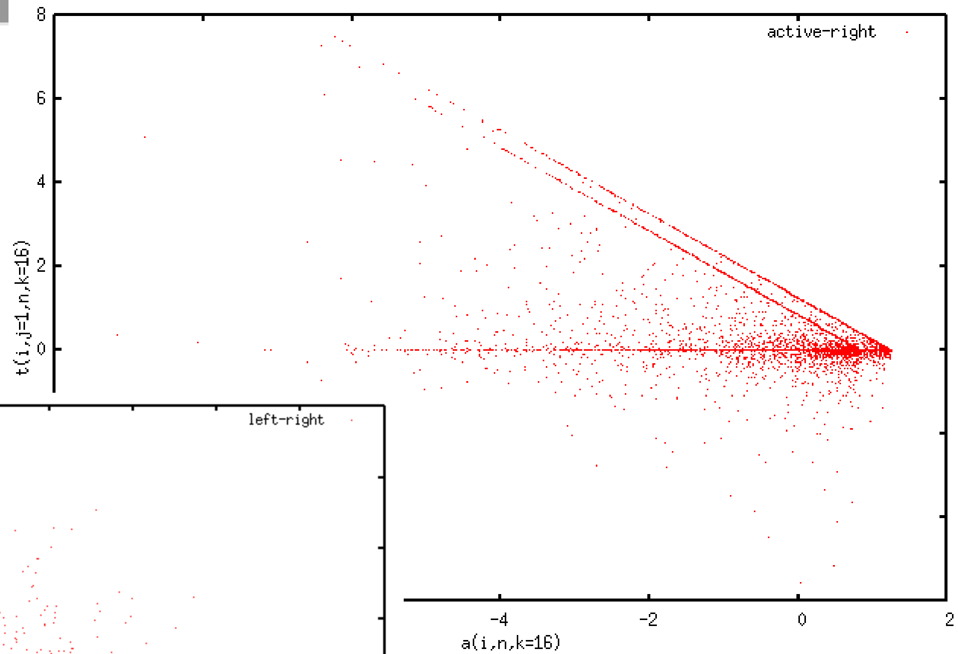


- Information storage peaks slightly within the ordered regime.
- Apparent transfer entropy slightly within chaotic regime.
- Complete transfer entropy continues to rise in chaotic regime.

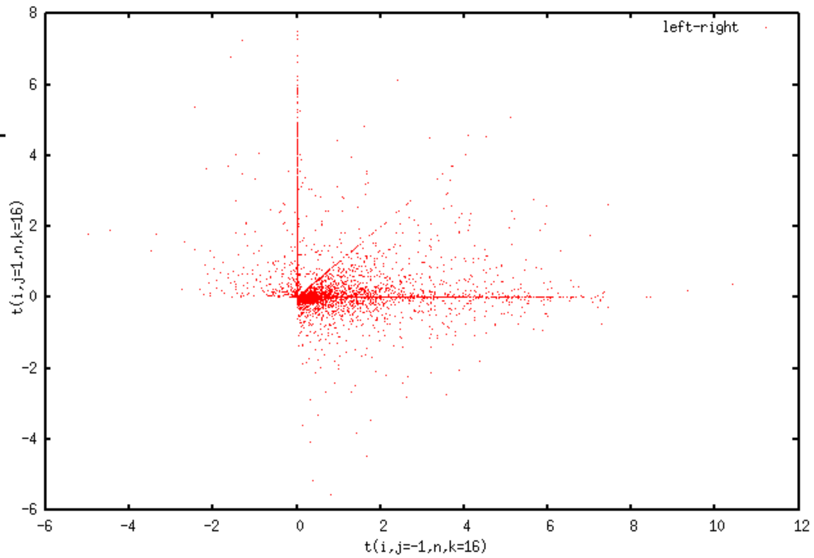
Structure between local measures in rule 110



t(i,j=-1,n) vs a(i,n)



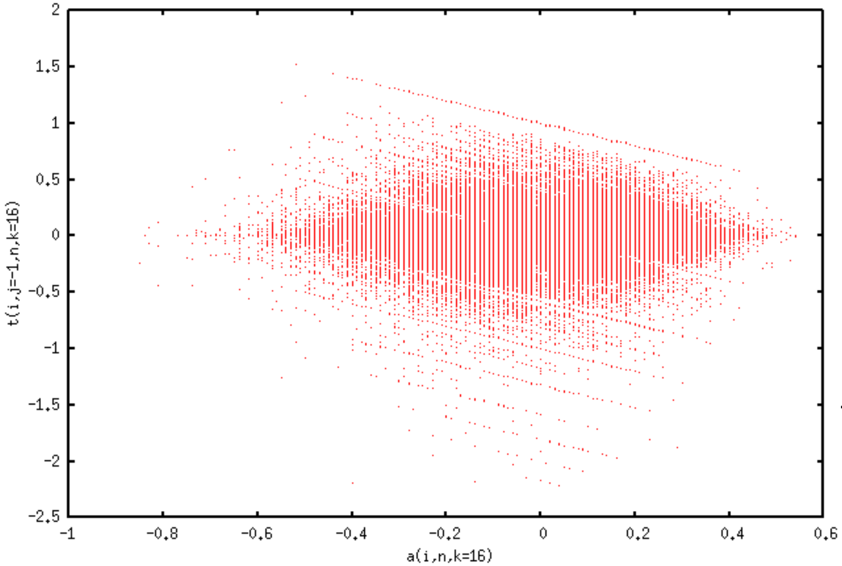
t(i,j=+1,n) vs a(i,n)



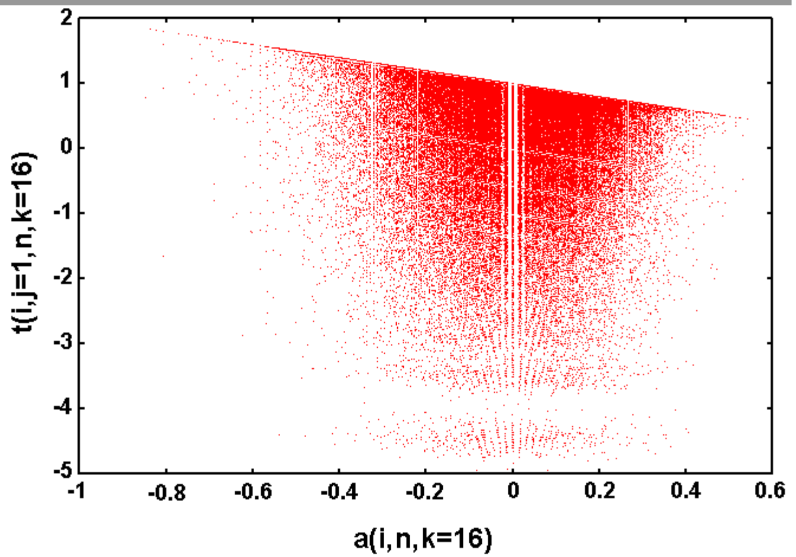
t(i,j=+1,n) vs t(i,j=-1,n)

- Rule **110** – clear structure seen in local information dynamics.
  - Structure appears to imply coherence of computation.
- What about other rules?

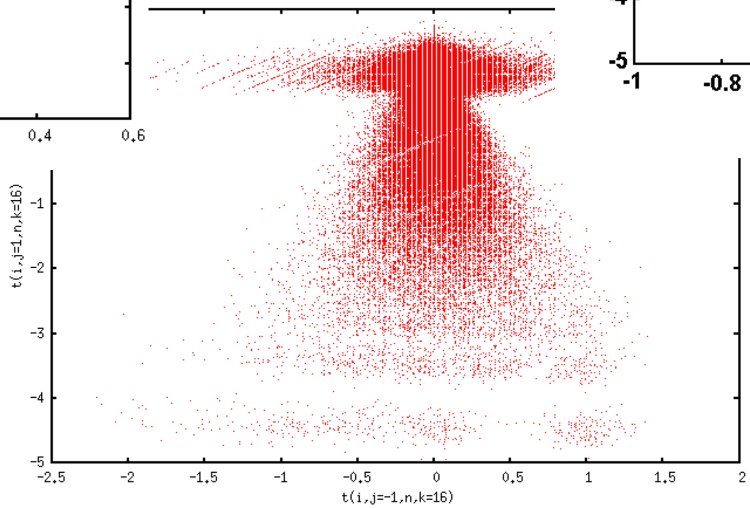
No structure between local measures in rule 30



t(i,j=-1,n) vs a(i,n)



t(i,j=+1,n) vs a(i,n)



t(i,j=+1,n) vs t(i,j=-1,n)

- Rule **30** – had no structure seen in local information dynamics.
- What do we expect for rule 22?

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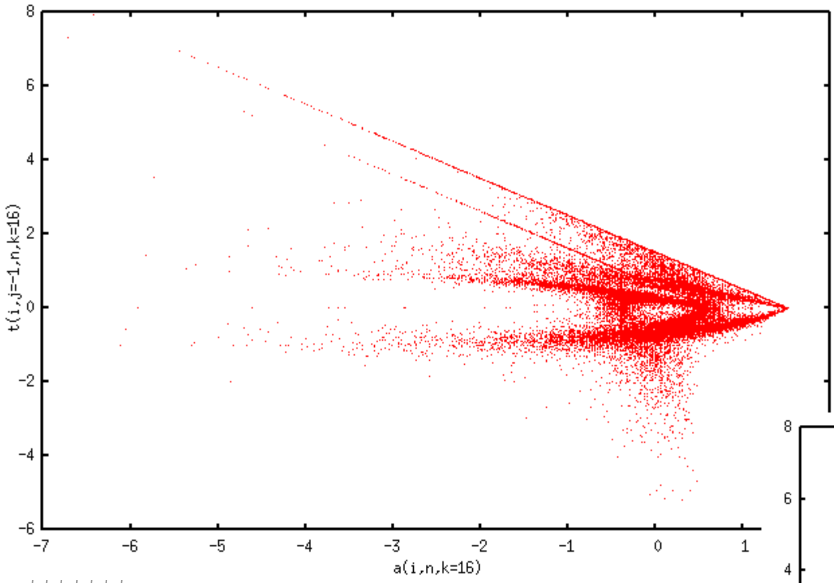
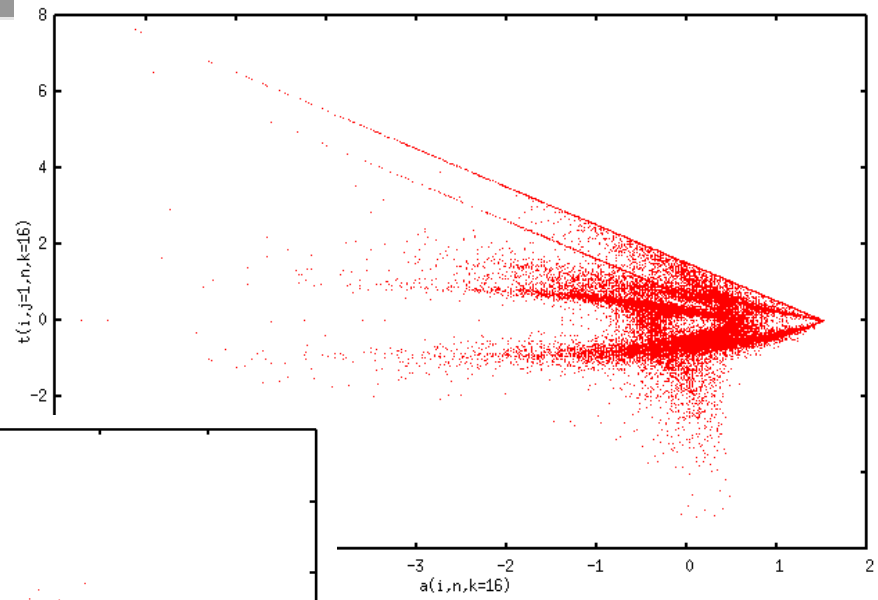
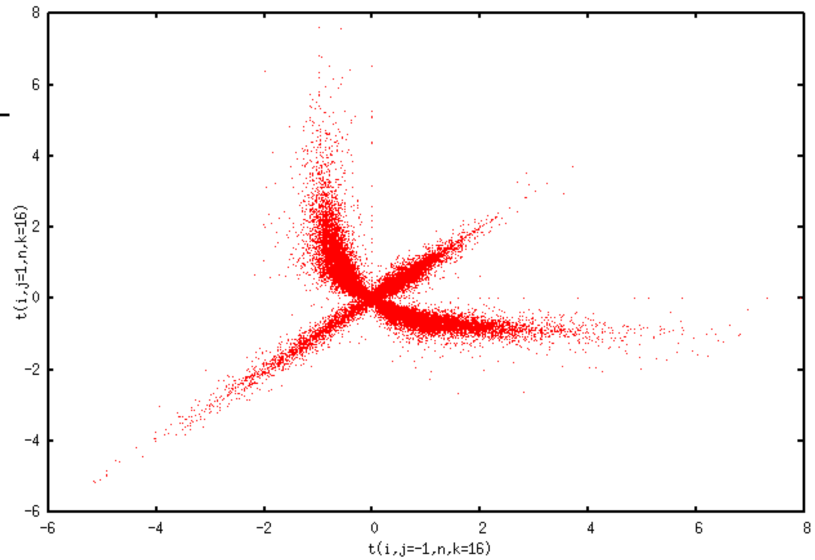
Structure between local measures in rule 22



t(i,j=-1,n) vs a(i,n)



t(i,j=+1,n) vs a(i,n)



t(i,j=+1,n) vs t(i,j=-1,n)

- Rule 22 – has structure in **between** the local dynamics which was not obvious from their individual profiles!
- These two views from the same framework provide new insights into debate on the nature of rule 22.

Conclusion

- Presented a framework for local information dynamics in terms of:
  - Information storage
  - Information transfer
  - Information modification
- Averages can be used to characterise computation
- Local view highlights coherent structure in complex computation and provides evidence for conjectures.
  - Can reveal hidden structure also
- Later/Future work:
  - Apply measures to biological systems (e.g. brain imaging).
  - Investigate relationship between network structure and dynamics
  - Guiding self-organisation with coherent computation.
  - How do info dynamics correspond to computational classes.

References

- These and other papers at http://www.cs.usyd.edu.au/~jlizier

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Thank you