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Coherent information structure in complex computation

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Abstract We have recently presented a framework for the information dynamics of distributed computation that locally identifies the component operations of information storage, transfer and modification. We have observed that while these component operations exist to some extent in all types of computation, complex computation is distinguished in having *coherent structure* in its local information dynamics profiles. In this paper, we *conjecture* that *coherent information structure* is a defining feature of complex computation, particularly in biological systems or artificially evolved computation that solves human-understandable tasks. We present a methodology for studying coherent information structure, consisting of state-space diagrams of the local information dynamics and a measure of structure in these diagrams. The methodology identifies both clear and "hidden" coherent structure in complex computation, most notably reconciling conflicting interpretations of the complexity of the Elementary Cellular Automata (ECA) rule 22.

Keywords complex systems · coherent structure · information structure · emergence · information theory · information transfer · transfer entropy · information storage · cellular automata · self-organization

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1 Introduction

Coherent information structure is a feature that is consistently observed in complex computation. In particular it appears that *nature evolves* coherent computation. Illustrative examples include: coherent signaling cascades providing information transfer in gene networks [9,10]; coherent waves of directional change in flocking birds [3] which are also referred to as "information cascades" in schooling fish [5]; the dynamic process of opening and closing stomatal apertures in plants via coherent collective waves of activity [34]; and coherent wave structures in neural computation [11]. Indeed, we observe that coherent computational structure also emerges from evolution in *artificial* systems, e.g.: the ϕ_{par} cellular automata rule which was evolved to solve the density classification problem and did so by using coherent glider structures [33,32]; and the emergence of coherent wave-like information structures in a snake-like robot evolved to maximize information transfer between its segments [23].

Interestingly, all of the above examples of coherent structure are analogous to *particle* or *glider* structures in cellular automata (CAs), the focus of our work here. CAs (e.g. see [41]) are discrete dynamical lattice systems involving an array of cells which synchronously update their states as a homogeneous deterministic function of the states of their local neighbors. Of particular importance here are Elementary CAs (ECAs), which consist of a one-dimensional array of cells with binary states, with each updated as a function of the previous states of themselves and one neighbor either side (i.e. neighborhood size 3 or range $r = 1$). CAs are widely used to study complex computation, since certain rules (e.g. ECA rules 110 and 54 - see [41] regarding the numbering scheme) exhibit emergent coherent structures which are not discernible from their microscopic update functions but which provide the basis for understanding the macroscopic computations being carried out [31]. These coherent structures traveling against a background *domain* region are known as *particles*.[1] Regular or periodic particles are known as *gliders*, while stationary gliders are known as *blinkers*. Particles are also often referred to as *domain walls*, since they are discontinuities in the background domain pattern. We focus on CAs in this paper because these coherent structures are well-identified, and there exists a reasonably good qualitative understanding of what is and what is not complex computation therein.

It has been widely observed that the known complex ECA rules (54 and 110) display the largest amount of coherent structure in terms of particles, in contrast with rules displaying ordered or chaotic behavior. This has been particularly well-demonstrated by the application of various information-theoretical measures to produce space-time profiles of the CAs which locally highlight these structures [39,16,15]. Similarly, we have presented a framework for the *information dynamics* of distributed computation, which is novel in separately quantifying information storage, transfer and modification [26–28,21,24]. Individual space-time profiles of these computational operations have been shown to quantitatively separate different emergent coherent structures into their computational roles: blinkers and regular domains implement information storage, moving particles facilitate information transfer, and particle collisions are non-trivial information modification events (see Fig. 1 and description in Section 2.2). These demonstrations suggest that these particles are coherent *information* structures. The inference of coherence, based on observation of close space-time points sharing similar information values, remains qualitative however.

Furthermore, we have demonstrated maximizations of these information dynamics near the critical state in order-chaos phase transitions in random Boolean networks (RBNs) [25]. RBNs are a generic model of discrete dynamical networks, and are known to display a phase transition from ordered to chaotic dynamics with respect to average connectivity or activity level. In that work, we suggested our results could be interpreted as a maximum capacity for coherent computation near the critical state.

[1] See also [39] for a discussion of the term "coherent structure" referring to particles (including blinkers) in this context.

On the basis of the above observations, we *conjecture* that the *coherence* of local information structure is a defining feature of complex computation, particularly evolved computation which solves human-understandable tasks. "Coherence" implies a property of sticking together or a logical relationship [1]: in this context we use the term to qualitatively describe *a logical spatiotemporal relationship between values in local information dynamics profiles*. For example, the manner in which particles in CAs give rise to similar values of local information transfer amongst spatiotemporal neighbors is coherent in this sense. We emphasize it is the *information structure* that is coherent here: the original CA states (from which the presence of particles is not clear) are not always obviously coherent themselves.

Using language reminiscent of Langton's analysis [20], we suggest that complex systems exhibit very *highly-structured coherent* computation in comparison to:

a. ordered systems, which exhibit coherence but minimal structure in a computation dominated by information storage or non-interacting transfer structures; and
b. chaotic systems, whose computations are dominated by rampant information transfer eroding any coherence.

Coherent structure is *useful* in complex computation because it provides *stable* mechanisms for storing information, transferring information, and modifying that information when required. Presumably it emerges in evolution of complex computation because of this utility. For these reasons, we suggest that coherent information structure may be a useful intrinsic goal in the domain of *guided self-organization* [35,36]. Evolution of coherent information structure could be particularly useful where task-based evolution faces initially flat task-based fitness landscapes, perhaps serving as a platform from which to launch better-equipped task-based evolution. Furthermore, coherent information structure is associated with computation that appears human-interpretable (e.g. the ϕ_{par} CA), which is an important trait for both acceptance and maintenance of self-organized systems in real-world deployments.

Our *goal* then is to measure the coherence of computational or information structure in a given system from its local information dynamics. That is, given a system $\mathbf{X}$ (i.e. with a time series $x_{i,n}$ for each variable X_i in the system, and a set of causal connections between them), we wish to compute a measure of the coherence of information structure in it. The approach is intended to first compute the local information dynamics at each space-time point in the system (resulting in the individual profiles shown in Section 2). The next step is to compute a measure of coherent information structure from these profiles. Existing measures for coherent structure are generally problem-specific and consider only contiguous clusters of similar values. A typical example is [17] which uses a mix of thresholding and spatiotemporal clustering of resulting binary values to identify specific coherent structures. Here however, we seek a general measure portable between different types of computation. Certainly the examination of local information dynamics values provides a system-independent perspective. Additionally, measuring coherence here must take into account the *continuous* nature of the measures, the fact that there are *multiple* information dynamics measures, and be *generic* in detecting coherent structure as relationships *between* these measures that may be more subtle than a contiguous cluster of similar values in a profile of one of them.

We also note that the goal of measuring coherent information structure is distinct from attempting to measure complexity itself (e.g. with the statistical complexity [6,39]), since it is a particular concept or system property. Indeed, it is unclear whether overall complexity and coherent computational structure will have a one-to-one correspondence. Systems could display coherent structure without complex computation, e.g. by exhibiting fixed or moving structures that do not interact with eachother. Also, guiding self-organization towards generally complex computation may not produce the coherent structure that we argued above to be useful. Furthermore, there are examples of computation argued to be complex from certain perspectives despite the apparent absence of local coherent structure. A prominent example is ECA rule 22 (see arguments in favor of its complexity in Section 2.2), where these conflicting perspectives are yet to be reconciled. Put simply, the concept of coherent information structure is worth exploring in its own right. On a related note though, we will also examine whether measuring coherent information structure provides useful insights into the differences in complexity of various computations.

As outlined above, we would expect the measure to be large in known complex computation. We emphasize though that while this measure is not expected to produce a *universal* complexity-entropy curve (as per [8]), it could be expected to be maximized with known complex behavior.

In this paper, we address our goal of measuring coherent information structure by focussing on CAs. We begin in Section 2 by describing our framework for information dynamics, and the ability of its underlying measures to highlight coherent structure in local profiles of CAs. We show in Section 2.3 that while the averages of these information dynamics measures provide useful insights, they cannot be directly used to quantitatively infer the presence of coherent structure. In Section 3 then, we present and explore diagrams of the multi-dimensional state-space formed by the local information dynamics, in particular the local information storage values and local values of information transfer from each source to a destination. This perspective allows the most general, unbiased interpretation of coherent information structure as a logical relationship between the information dynamics, in alignment with our qualitative interpretation and requirements above. Importantly, we demonstrate these diagrams as a particularly useful tool in this context. We conclude that coherent computational structure should be measured as *a logical relationship between values in the local information dynamics state-space*, and present the measure I^{ss} for this purpose in Section 4. The state-space diagrams and the measure I^{ss} provide a methodology which identifies clear and "hidden" coherent structure in complex computation, in particular revealing previously hidden coherent structure in the information dynamics of rule 22 and reconciling the differing perspectives on how complex it is.

2 Local information dynamics

We have proposed a framework for the local information dynamics of distributed computation in [26,22, 27,28,21,24]. The framework describes computation in terms of information storage, transfer and modification at each spatiotemporal point in a complex system. Built on information theory (e.g. see [29]), it provides a common, non-linear, application-independent language in which to analyze and design complex systems.

2.1 Measures of information dynamics

Information storage refers to the amount of information in the past of a variable that is relevant to predicting its future. The *active information storage* measures the stored information that is *currently in use* in computing the next state of the variable [28,21]. Specifically, the *local active information storage* for X is the local (or un-averaged) mutual information between its semi-infinite past $x_n^{(k)} = \{x_{n-k+1}, \ldots, x_{n-1}, x_n\}$ (as $k \to \infty$) and its next state x_{n+1} at time step $n+1$:

$$a_X(n+1) = \lim_{k\to\infty} \log_2 \frac{p(x_n^{(k)}, x_{n+1})}{p(x_n^{(k)})p(x_{n+1})}. \quad (1)$$

The active information storage is the average over time: $A_X = \langle a_X(n) \rangle_n$. $A_X(k)$ and $a_X(n+1,k)$ represent finite-k estimates. Of course, we can write $a_X(n,k)$ with n representing the next time step instead of $n+1$ for notational convenience. Also, we write $a(i,n+1,k)$ to represent the local active information storage at time $n+1$ for element i in a lattice system.

Information transfer is formulated by the *transfer entropy* [38] (TE) as the information provided by a source about a destination's next state that was not contained in the past of the destination. Specifically, the *local transfer entropy* [26] from a source Y to a destination X is the local mutual information between the previous state of the source y_n and the next state of the destination x_{n+1}, *conditioned* on the semi-infinite past of the destination $x_n^{(k)}$ (as $k \to \infty$):

$$t_{Y\to X}(n+1) = \lim_{k\to\infty} \log_2 \frac{p(x_{n+1}|x_n^{(k)}, y_n)}{p(x_{n+1}|x_n^{(k)})}. \quad (2)$$

Again, the transfer entropy is the (time) average $T_{Y\to X} = \langle t_{Y\to X}(n)\rangle$, while finite-$k$ estimates are represented as $T_{Y\to X}(k)$ and $t_{Y\to X}(n+1,k)$. Also, we write $t(i,j,n+1,k)$ to represent the local transfer entropy at time $n+1$ to element i from element $i-j$ (i.e. across j elements) in a lattice system. The TE can be measured for any two time series, but only represents information transfer when measured on a causal link [22]. Importantly, the TE properly measures a directed, dynamic flow of information, unlike previous inferences with the mutual information which measure correlations only.

The TE can also be formulated to condition on the states $\mathbf{v}_{x,n}^y$ of all *causal information contributors* to the destination (the set $\mathbf{V}_X$) except the source Y, so as to completely account for the contribution of Y. This gives the *complete* transfer entropy [26], with local values defined as:

$$t_{Y\to X}^c(n+1) = \lim_{k\to\infty} \log_2 \frac{p(x_{n+1}|x_n^{(k)}, y_n, \mathbf{v}_{x,n}^y)}{p(x_{n+1}|x_n^{(k)}, \mathbf{v}_{x,n}^y)}, \quad (3)$$

$$\mathbf{v}_{x,n}^y = \{z_n | \forall Z \in \mathbf{V}_X, Z \neq Y\}. \quad (4)$$

Again, the complete TE is the time average $T_{Y\to X}^c = \langle t_{Y\to X}^c(n)\rangle$, finite-$k$ estimates are represented as $T_{Y\to X}^c(k)$ and $t_{Y\to X}^c(n+1,k)$, and we write $t^c(i,j,n+1,k)$ in lattice systems. To indicate the contrast between the measures, the formulation in Eq. (2) is then labeled the *apparent* transfer entropy. Importantly, in accounting for the contribution of other sources the complete TE can detect *interaction-based transfer*. This is where the outcome of a computation is not predictable without analyzing all of the input sources, e.g. in an XOR (Exclusive-OR) operation. The apparent TE is useful in its own right, because it can only be large where the source has a *coherent* effect on the destination (without requiring interaction with other sources).

We note that the information (or local entropy) $h(i,n+1) = -\log_2 p(x_{i,n+1})$ required to predict the next state of a destination at time step $n+1$ can be decomposed as a sum of [27,22] the active information storage, (incrementally conditioned) transfer entropies, and remaining intrinsic uncertainty. In ECAs we have no intrinsic uncertainty, and the TE terms are the apparent TE from one neighbor plus the complete TE from the other (using lattice notation):

$$h(i,n+1) = a(i,n+1,k) + t(i,j=-1,n+1,k) + t^c(i,j=1,n+1,k), \quad (5)$$

or vice-versa in $j=1$ and -1.

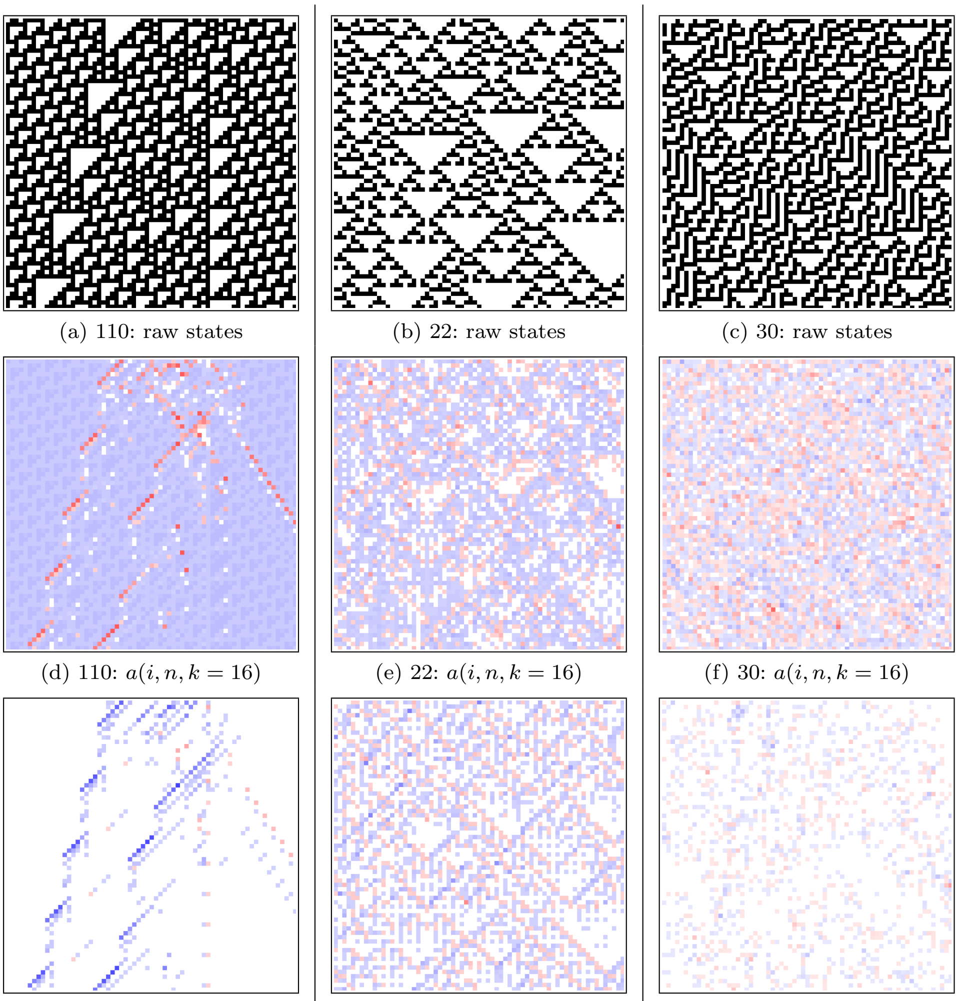
Finally, we note that measurement of **information modification** using the separable information is described in [27], but not directly used in this paper.

2.2 Local information dynamics in CAs

The measures of the framework have been applied to CAs in [26–28,21,24]. As demonstrated for complex ECA rule 110 in Fig. 1, this application quantified blinkers and regular domains as the dominant information storage elements, particles (gliders and domain walls) as the dominant information transfer agents, and (not shown in Fig. 1) particle collisions as the dominant (non-trivial) information modification events. These results align with existing conjecture on the nature of distributed computation in CAs (e.g. [20,33,31]).

Importantly, the results highlight each of these particles as *coherent information structures*. Specifically, gliders are coherent information transfer structures and blinkers are coherent information storage entities, since spatiotemporally neighboring (or close) points in these structures have similarly high values of transfer and storage respectively.

Fig. 1 also plots the local profiles generated by these measures for ECA rule 22. There has been a long-standing debate regarding the nature of computation in this rule. Some authors consider it to be complex, citing the difficulty in estimating its temporal entropy rate [13,12], its ability to produce the self-similar structure of the "Sierpinski Gasket" [41], that it is a 1D mapping of the 2D Game of Life CA (known to have the capability for universal computation [4]) [30], and complex structure in the language it generates [2]. Also, we report here that we have evaluated the C_1 complexity measure defined in [19] (an enhanced version of the variance of the input entropy [42]) for all ECAs. We found rule 22 to clearly exhibit the largest value of this measure (0.78 bits to rule 110's 0.085 bits). Similarly, we evaluated the statistical complexity [6,39] and found it to be larger



(a) 110: raw states (b) 22: raw states (c) 30: raw states

(d) 110: $a(i, n, k = 16)$ (e) 22: $a(i, n, k = 16)$ (f) 30: $a(i, n, k = 16)$

(g) 110: $t(i, j = -1, n, k = 16)$ (h) 22: $t(i, j = -1, n, k = 16)$ (i) 30: $t(i, j = -1, n, k = 16)$

Fig. 1 Local information dynamics in ECA rule 110, rule 22, and rule 30: 67 time steps for 67 cells for each CA, with time increasing down the page. Cells are colored blue for positive values and red for negative. Raw states in (a), (b) and (c); Local active information storage in (d) (max 1.22 bits, min. -9.21 bits), (e) (max. 1.51 bits, min. -8.17 bits) and (f) (max. 0.54 bits, min. -0.87 bits); Local transfer entropy one cell to the left in (g) (max. 10.43 bits, min. -6.01 bits), (h) (max. 9.68 bits, min. -7.10 bits) and (i) (max. 1.81 bits, min. -2.74 bits).

for rule 22 than known complex rules 54 and 110 (at 4.22, 3.80 and 3.91 bits respectively).[2]

Other authors suggest that it is chaotic rather than complex, citing its high sensitivity to initial conditions and perturbations [41, 12], fast statistical convergence [14], and exponentially long and thin transients in state space [42].

Returning to Fig. 1, while there is certainly much activity with respect to each computational operation, these plots suggest that there is no coherent structure in the information dynamics profiles for rule 22. This aligns with the local profiles generated by other measures (e.g. [39]) which have not identified any particles for this rule. It also aligns with the apparent absence of coherent structure for other chaotic rules, e.g. rule 30 (see Fig. 1).

Certainly, the profiles in Fig. 1 appear to identify coherent information structure in terms of particles where they occur. Furthermore, they suggest (in alignment with previous filtering work) that the known complex rules (rule 110 shown here, and 54 shown in [27, 21]) contain the largest amount of such coherent information structure in their profiles. Rule 18 (see [26, 21]) contains a smaller amount of less coherent structure in its domain walls. Chaotic rules 22 and 30 (shown here) certainly exhibit all of the elementary functions of computation, but do not appear to contain any coherent structure to their computations.

[2] Measured using the CimulA package [37] over 600 time steps of 100 000 cells, with light cone depths of 3 time steps.

Table 1 Table of average information dynamics (all with $k = 16$, and values to 2 decimal places), for several ECA rules. Units are in bits.

Rule	H	H_μ	A	$T_{j=1}$	$T_{j=-1}$	$T^c_{j=1}$	$T^c_{j=-1}$
110	0.99	0.18	0.81	0.07	0.11	0.07	0.11
54	1.00	0.27	0.73	0.08	0.08	0.19	0.19
22	0.93	0.75	0.19	0.19	0.19	0.56	0.56
18	0.82	0.53	0.29	0.01	0.01	0.52	0.52
30	1.00	0.99	0.01	0.73	0.01	0.98	0.26

However, qualitative interpretation of coherent information structure from a spatiotemporal profile rather than a specific measure has the potential to be subjective, and cannot be applied to non-lattice systems without a spatial layout. We thus focus on directly measuring such structure.

2.3 Average information dynamics in CAs

The ability to locally highlight particles in CAs above suggests that the averages of the information dynamics may be candidates for measuring coherent information structure. There are additional reasons for this suggestion. For example, the apparent TE was described as measuring *coherent* effects of a single source on a destination (Section 2.1). Also, both the active information storage and apparent TE were shown to be maximized near the critical state in the phase transition in RBNs [25].

Here, we briefly investigate whether large average values of the information dynamics have a one-to-one correspondence with the existence of coherent structures and complex computation. This may be the case in *certain* phase transitions in a single parameter; other systems may exhibit increased likelihoods of their co-existence (e.g. in RBNs). However, we are interested in whether a correspondence holds in general, e.g. in CAs where there is no known continuous phase transition in a single parameter. The average measures for several important ECAs discussed previously were calculated (using 600 time steps for 10,000 cells starting from a random initial state) as shown in Table 1.

Certainly large values for the active information storage $A(k = 16)$ are observed for the known complex rules. Yet the major component of the storage in rules 54 and 110 come from their background domains [28, 21], and it is straightforward to conceive simple oscillating systems maximizing this measure without any interacting coherent structures.

Also, the apparent TE cannot measure coherent structure on its own since the chaotic rules 22 and 30 have significantly larger values than the complex rules. To some extent, one could suggest that the complex rules exhibit the apparent TE in *each* channel j as a large proportion of the complete TE $T^c(j, k = 16)$ for that channel. As described in Section 2.1, apparent TE can only be high where the source has a clear coherent influence on the destination, while complete TE can additionally be high due to *interaction-based transfer*. In the complex CAs, single sources can often influence destinations without needing to interact with other sources, supporting the movement of coherent particle structures and propagation of coherent effects over long distances. Importantly, this occurs for multiple channels, meaning that we have bidirectional traveling coherent structures that should *interact* at some point. On the other hand, the chaotic CAs appear to have their dynamics dominated by multi-source interactions rather than single-source effects, eroding the coherence of computation.[3] Importantly though, these observations quantify neither coherence nor complexity of computation, and again one can conceive of simple systems which produce similar indicators without having coherent interacting structures.

Finally, our interpretation of coherence as meaning a logical spatiotemporal relationship between local values suggests that it may be measured via the autocorrelation within profiles of each of their local information dynamics. For example, one could autocorrelate TE profiles across the same spatial distance and time step that the TE profile was calculated for. This suggestion was explored in [21]. It was found that the autocorrelation was a useful heuristic, similar to the average values discussed above. However, it still treats the information dynamics independently which misses coherent structure in their interaction. It is also a linear measure, which could miss more subtle types of relationships here. Furthermore, it is difficult to extend beyond lattice systems (say to RBNs) where agents are heterogeneous and there are no generic spatial relationships such as “1 step to the right”.

3 Local information dynamics state-space

As per our qualitative description in Section 1, coherence may be broadly interpreted as a logical relationship *between* the individual local information dynamics measures rather than only within individual profiles alone. Indeed, using a more broad definition allows

[3] Similar arguments are derived from average values of the separable information in [21], where collisions or non-trivial information modification events are observed to be frequent in chaotic dynamics (disturbing coherent computation) and comparatively rare in complex dynamics where they can have high impact in coherent computation.

an unbiased measurement of coherent structure. To explore relationships as per this broad definition, in this section we investigate information state-space diagrams which plot the local information dynamics *against* each other. We demonstrate that these diagrams are useful tools in revealing both clear and hidden coherent information structure in the computation in CAs.

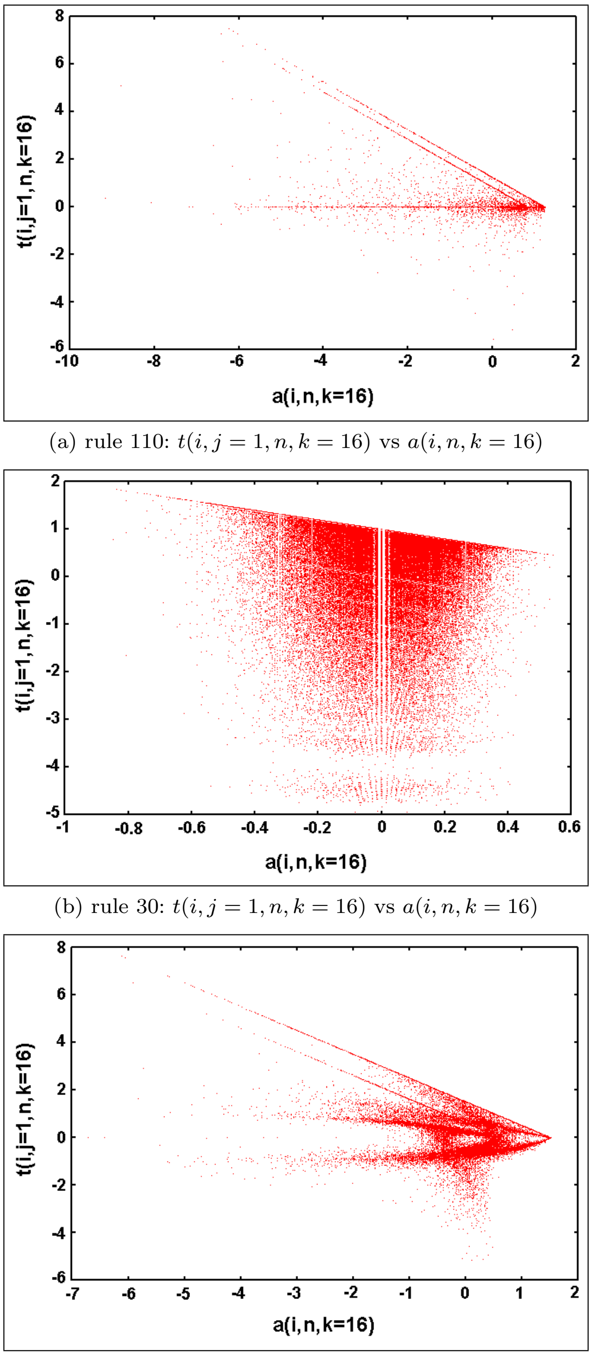
The state-space referred to here is the multi-dimensional information space consisting of the local values of each of the individual information dynamics. In theory, plots of this state-space should reveal any logical relationship between the individual measures, and by our broad definition such relationships embody coherent information structure. Fig. 2 projects the multi-dimensional information state-space onto two-dimensional diagrams. There we plot the local apparent transfer entropy $t(i, j = 1, n, k = 16)$ versus local active information storage $a(i, n, k = 16)$, for several CA rules. Each point in these diagrams represents the local values of the measures at one spatiotemporal point (i, n). We emphasize that coherent *spatio*temporal structure can be captured in these diagrams since the TE measurements consider neighboring values across space and the temporal history of the destination.

Similar state-space diagrams are known to provide insights into structure that are not visible when examining either measure in isolation. An example is examining structure in classes of systems (such as logistic maps) by plotting *average* excess entropy versus entropy rate while changing a system parameter [8]. In contrast here we are looking at structure *within* a single system rather than across a class of systems.

As could be expected, **the state-space diagrams for rule 110 exhibit interesting structure** (Fig. 2a) with significant clustering around certain areas and lines, reflecting its status as a complex rule. The two diagonal lines in Fig. 2a are upper limits representing the boundary condition $t^c(i, j = -1, n, k = 16) \geq 0$ [26] for both destination states "0" and "1". That is, via Eq. (5) we have:

$$h(i, n+1) \geq a(i, n+1, k) + t(i, j = 1, n+1, k), \qquad (6)$$

for both the upper limits $h(i, n+1) = -\log_2 p(x_{i,n+1} = 0)$ and $-\log_2 p(x_{i,n+1} = 1)$. The horizontal line is for $t(i, j = 1, n, k = 16) = 0$. Rather than being merely boundaries these lines are structural elements in their own right, being dense areas of the state-space and adding significant structure to it. Their long extent suggests a large variety of coherent activity, because with zero transfer from one source and $a(i, n+1, k)$ much less than 0 the other source has a greater opportunity to exert a strong coherent influence on the destination.



(a) rule 110: $t(i, j = 1, n, k = 16)$ vs $a(i, n, k = 16)$

(b) rule 30: $t(i, j = 1, n, k = 16)$ vs $a(i, n, k = 16)$

(c) rule 22: $t(i, j = 1, n, k = 16)$ vs $a(i, n, k = 16)$

Fig. 2 State space diagrams of local transfer entropy (one step to the right) $t(i, j = 1, n, k = 16)$ and local active information $a(i, n, k = 16)$ at the same space-time points (i, n) for ECA rules 110, 30 and 22.

We then consider the chaotic rule 30 whose raw states and local information dynamics profiles exhibit an apparent absence of coherent structure. In contrast to the results for rule 110, **the state-space diagrams for rule 30 exhibit minimal structure** (Fig. 2b), with a smooth spread of points across the space reflecting its underlying chaotic nature. The same mathematical limits exist of course but are not a dense, individual part of the structure as for rule 110.

As shown in Fig. 1, rule 22 also exhibits an apparent absence of coherent structure in its individual space-time information profiles, similar to rule 30. As such, one may expect state-space diagrams for rule 22 to exhibit a similar absence of structure. However Fig. 2c shows that this is not the case: **the state-space diagrams for rule 22 exhibit significant structure**, with similar clustering to that of rule 110. Indeed, rule 22 exhibits similar linear structures to rule 110, which as argued above suggests the potential for strong coherent influence of a source on a destination.

This is a *highly significant result*, because **local information dynamics is the first approach able to reconcile the opposing views of rule 22 as complex and chaotic**.[4] When the local information dynamics profiles are viewed *individually* (Fig. 1), no coherent structure is revealed. This is in alignment with local information profiles from other authors, e.g. [39]. In contrast, here we see that when plotted *together* in state-space diagrams we see an indication of a coherent relationship *between* the local information dynamics. This view lends credence to the claims of complex behavior for rule 22 discussed in Section 2.2. Its coherent information structure is not complex enough to be *obviously* revealed to the eye by the individual profiles, and is perhaps very subtle.[5] Indeed, this structure may underpin coherent computation at other scales.

[4] Indeed, the candidate measures considered in Section 2.3 did not capture its alignment with the known complex rules in this respect.

[5] On close inspection of the local information transfer profile for rule 22 in Fig. 1h, there do appear to be some weak coherent information transfer structures corresponding to the diagonal edges of the large white triangles in the raw states in Fig. 1b. They are weak or subtle in comparison to the gliders in rule 110 since they do not stand out against other information transfer in the background. That said, it is possibly these structures which are being captured as the coherent information structure by our analysis of rule 22 here. This would be interesting, since these triangular structures in rule 22 are related to the fractality of the "Sierpinski Gasket" that it can produce [41].

4 Measuring coherent information structure in the state-space

We conclude that **coherent information structure should be quantified as the logical relationship between values in the local information dynamics state-space**. This is because these diagrams satisfy the requirements described in Section 1 and have been demonstrated in Section 3 to identify both clear and hidden structure. In this section, we discuss how to quantify coherent information structure in this manner and present the measure I^{ss} for this purpose. Subsequently, we apply the measure I^{ss} to CAs and discuss the results.

To remain maximally unbiased, the measurement should be made in the underlying multi-dimensional information state-space rather than the two-dimensional projections plotted in Fig. 2. The primary question is which of the information dynamics should be included in the measure. Certainly, all of the information sources for a given destination must be represented. One could do so by by examining the state-space made up of $a_X(n,k)$ and the incrementally conditioned mutual information terms in decompositions such as Eq. (5). However, this approach leaves ambiguity in the order of considering sources in the incrementally conditioned mutual information terms. Also, as more sources are conditioned on, more redundant information is built into this state-space about the previously considered sources.[6] A more appropriate approach without such ambiguity or inbuilt bias is to consider the state-space of measures of information storage and apparent transfer from each source, i.e. $a_X(n,k)$ and $t_{Y\rightarrow X}(n,k)$ for all sources Y.

The next question is how to measure structure in the state-space of these measures. Certainly, measuring structure in two or more dimensional patterns is known to be particularly difficult [7]. We select the multi-information (or integration, see [40]) for this purpose, since as a generalization of the mutual information to a larger set of variables, it measures the degree of dependence between these variables. This aligns well with our intention to measure the *logical relationship* between the information dynamics variables.

We propose the **state-space multi-information** as a measure of *coherent information structure* at an information destination X due to causal sources $\{\mathbf{V}_X \setminus X\}$: $Y_1, Y_2, ..., Y_G$ as the multi-information I^{ss}_X between its active information storage and the apparent transfer

[6] For example, $t^c(i, j=-1, n, k)$ is almost completely specified by $a(i,n,k)$ and $t(i, j=1, n, k)$ in ECAs, except for any difference in $h(i,n)$ between the "0" and "1" states.

Table 2 Table of the multi-information $I^{ss}(k=16)$ in the local information state-space in ECAs.

ECA rule	$I^{ss}(k=16)$ (bits)
110	0.50
54	0.95
22	0.72
18	0.15
30	0.11

entropies to it:

$$I_X^{ss} = \lim_{k\to\infty} I_X^{ss}(k), \quad (7)$$

$$I_X^{ss}(k) = I\left(a_X(n,k); t_{Y_1\to X}(n,k); \ldots ; t_{Y_G\to X}(n,k)\right), \quad (8)$$

$$= H\left(a_X(n,k)\right) + \left(\sum_{g=1}^{G} H\left(t_{Y_g\to X}(n,k)\right)\right) - H\left(a_X(n,k), t_{Y_1\to X}(n,k), \ldots, t_{Y_G\to X}(n,k)\right). \quad (9)$$

For a lattice system we can write $I^{ss}(i,k)$ for agent i; where the agents are homogeneous (e.g. CAs) we can estimate the PDFs over all observations in the system, and average over all agents to get $I^{ss}(k) = \langle I^{ss}(i,k)\rangle_i$. Initially, it may seem strange to make an information-theoretic measure of information-theoretical values; however there is no reason that structure in the information state-space should not be measured information-theoretically.

4.1 Coherent information structure measurements in CAs

The state-space multi-information $I^{ss}(k=16)$ was measured for the CA rules investigated in previous chapters. These measurements used 200 000 sample points, with kernel estimation of the underlying entropies (see [18,38]) since the sample points have continuous values. These estimates use a step kernel with precision $r = 1.0$ bits and maximum distance norms. The results are displayed in Table 2.

The measurements of coherent information structure using $I^{ss}(k=16)$ align to a large degree with our qualitative observations of these rules above. We find that rules 110 and 54 contain strong relationships between the information dynamics in their state-space. In contrast, rule 30 contains very little coherent structure. We also measure only a small amount of coherent structure for rule 18, since the structure in its domain walls is only a relatively minor part of the dynamics compared to its background domain. While there is a pattern to this domain, as we have observed there is almost no relationship between the interacting sources (see Section 2.2), so the lack of coherent structure here is not surprising. As per our qualitative observations above, significant coherent structure is also measured for rule 22. This confirms that the framework can resolve the conflicting views of rule 22 as complex and chaotic. We remind the reader that this measure is not of complexity itself and, given the subtle nature of structure of the coherent structure in rule 22 (not being revealed in *individual* information dynamics profiles), we cannot conclude that its behavior is any more complex than that of rules 110 and 54.

We also note that the measure returns large values for CA rules (e.g. 0.90 bits for rule 47) which contain simple gliders in a single channel j only. Since these gliders never interact with other gliders, these rules are not viewed as complex. The large measurement is not incorrect though, since the measure is genuinely detecting coherent structure. We note that this structure only manifests in a single two-dimensional projection of the state-space ($t(i,j,n,k)$ versus $a(i,n,k)$ for the channel j the gliders travel in). It is undesirable to infer as much coherent structure to these types of dynamics as for rules such as 110 which have structure in all dimensions of the state-space. We expect that new approaches addressing the acknowledged difficulties in defining measures of multi-dimensional structure (e.g. see [7]) may provide improvements over the multi-information for measuring structure in the local information state-space here.

Despite this minor limitation, these results (in particular for rule 22) underline that I^{ss} usefully captures clear and hidden coherent information structure in the state-space. We emphasize though, that the state-space diagrams themselves contain much more detail about the relationships between these axes of complexity than this averaged measure.[7]

5 Conclusion

We have conjectured that coherent information structure is a defining feature of complex distributed computation, particularly in biological systems. We have presented a methodology for exploring the coherence of such computation, using our framework for local information dynamics. The methodology focuses on state-space diagrams of the local information dynamics, since these provide the most unbiased approach to detecting logical relationships between the local information dynamics values. These diagrams produce useful visual

[7] This is similar to the manner in which the local information dynamics measures themselves reveal more about the underlying computation than their averages do.

insights and indeed we define coherent information structure as the logical relationship between values in the local information dynamics state-space. We proposed the multi-information I^{ss} in this state-space as a measure for this concept.

We applied the methodology to cellular automata, a classical complex system. The application showed that the method captures not only clear but also hidden coherent structure in the underlying computation. A prominent example here is ECA rule 22, where no coherent structure is visible in spatiotemporal profiles of the individual information dynamics, but is revealed in the state-space diagrams between them and quantified by our measure I^{ss}. This is a significant result, since our framework is the first approach which can reconcile the two conflicting views of rule 22 as complex and chaotic. While we emphasise that I^{ss} is not intended to measure complexity as a whole, this result provides important evidence that coherent information structure is a key feature of complex computation.

We intend to explore the relationship of coherent information structure to other properties of complex computation, and indeed other measures of complexity. In particular, further work is required to establish the meaning of the relationship between the information dynamics measures in rule 22. We also intend to explore the application of the measure I^{ss} for the analysis of other systems, e.g. RBNs [21]. Furthermore, we shall examine the application of new measures of multi-dimensional structure to the information state-space as they are published, in order to check if they can address the minor limitations of I^{ss} described in Section 4.1. Finally, we intend to explore the utility of the measure of coherent information structure I^{ss} as an intrinsic goal in *guided self-organisation*. In this domain, guiding towards higher values of I^{ss} has the potential to generate coherent computational structure which could be usefully applied to solve complex tasks.

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