Information dynamics in small-world Boolean networks

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**Introduction**

**Aim:** To study the *topological* phase transition in small-world networks from the perspective of their *dynamic* computational properties.

**Results:**
- Information storage dominates the regular/ordered regime
- Information transfer dominates the random/chaotic regime
- Small-world transition combines high information storage and transfer
Overview

1. Motivation
2. Random Boolean networks
3. Information dynamics
4. Small-world networks
5. Results and Discussion
Computation in small-world random Boolean networks: Motivation
Computation in small-world networks

The structure and generation of small-world networks are well-understood ... but what are the (computational) dynamical properties that make them so useful?

Topology gives rise to dynamics, but dynamics represents the specific action of a network.

Is there a maximisation of computational properties here, as per other networks between ordered and chaotic regimes?
Tools

We need two important tools here:

1. A model for the dynamics on these networks.
   We select *random Boolean network* (RBN) dynamics:
   - Generality as network models with a large sample space.
   - Classic model for GRNs.
   - Well-known order-chaos phase transition.
   - Other authors have used RBNs for this purpose
     -(e.g. Lu & Teuscher 2009, Zhang & Zhao 2009)

We select information dynamics:

- Studying how information is manipulated in intrinsic computation has the ability to compare results across different types of dynamics.
  - Barabasi, 2009: Diversity of dynamics processes has been a major roadblock to obtaining a common framework here.
- Information theoretic: model-free, captures nonlinear relationships
- Language of computation pervades descriptions of time-series dynamics in networks:
  - Mitchell 2006: “understanding the ways in which information spreads in networks is one of the most important open problems in science”.
- We have previously used it to analyse RBNs.
Random Boolean networks (RBNs)
RBNs generally have:

- N nodes in a directed structure
- Which is determined at random from an average in-degree $\langle K \rangle$

Each node has:

- Boolean states updated synchronously in discrete time
- Update table determined at random using bias $r$ for a “1” result
RBNs

Function

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time

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RBNs

Phase diagram

Ordered and chaotic phases in RBNs
Quantified using Hamming distance on response to perturbations
Information dynamics
... is the study of distributed computation in complex systems, in terms of 3 fundamental operations on information:
Information dynamics

- **Info Storage**: info in past of a variable relevant to predicting its future
- **Active Info Storage**: mutual info between past and next step:
  \[ A(k) = I(X^{(k)}; X') \]

Lizier et al, 2007-10

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Information dynamics  Total information

• We can write the total information to predict the next state of a destination in terms of these quantities:

\[ H_X = A_X + H_{\mu X} \]

Storage  
Transfer plus intrinsic uncertainty

\[ H_X = A_X + T_{Y_1 \rightarrow X} + \text{higher order terms} \]
Information dynamics Transfer

- **Apparent transfer entropy**: mutual information between source and destination conditioned on the past of the destination (Schreiber 2000, local TE: Lizier et al 2010), i.e.:

\[
T_{Y_1 \rightarrow X(k)} = I(Y_1; X' | X^{(k)})
\]
Information dynamics Locally in CAs

- Blinkers and domains are info storage entities.
- Gliders are coherent info transfer entities.
Small-world networks
Small-world networks

Overview

Regular  Small-world  Random

$\gamma = 0$  Increasing randomness  $\gamma = 1$

$C(p) / C(0)$  Path length

Watts & Strogatz, 1998
Small-world networks + random Boolean functions

3 parameters define our small-world RBNs:

1. $K$ represents average number of incoming links
   However for small-world networks we only define integer and half-integer $K$.

2. $\gamma$ represents the level of randomisation of the structure.
   (probability of rewiring source of each link; or destination in some experiments)

3. $r$ represents probability of a “1” outcome in all logic tables.
   Reminder: structure does not determine function alone!
Information dynamics in small-world random Boolean networks
Method
details, details ...

**Ensemble investigation**

- Lots of networks for each combination of parameters
- Taking averages of each measure across all nodes and sample networks
- RBNs modelled with enhancements to the RBNLab software (Gershenson 2003).
- Focussing on the transient (i.e. where attractor is **computed**).
Random networks verification

$\gamma = 1$

- Verifies 2D phase transition results for RBNs (in fully random networks)
  - Ordered phase dominated by information storage
  - Chaotic phase dominated by information transfer
  - Maximisations of $A_x$ on ordered side, $T_{Y \rightarrow X}$ on chaotic side.

Lizier et al, 2008
Small-world networks

Method

- Fix $\langle K \rangle = 4$, vary $r$ and $\gamma$
- Establish where small-world topological phase transition is:

  - (Topological) Small world region $\sim 0.03 \leq \gamma \leq 0.01$
Small-world networks  New phase transitions

$\langle K \rangle = 4$

- Verifies 2D phase transition results for small world RBNs
  - Get a phase transition for altering $\gamma$, $r$ or $K$ (not shown)
  - Introduction of local links makes network dynamics more ordered.
  - Location of phase transition has much similarity to small world regime, but is not the same
**Small-world networks**  

Information storage

\[ \langle K \rangle = 4 \]

- Large information storage for regular (unrandomised) networks
  - No maximisation in or near small-world regime
  - **Suggests info storage is strongly supported by clustering / community structure in regular networks**
  - Maximisation of info storage versus \( r \) has its position shifted with \( \gamma \).
Small-world networks  Information storage

$\langle K \rangle = 4$

For destination randomisation
Small-world networks

Information transfer

\[ \langle K \rangle = 4 \]

- Large information transfer for randomised networks
  - Suggests info transfer is supported by introduction of long links as network is randomised

- Info storage dominates ordered side and info transfer dominates chaotic side of small-world transition
  - The small-world phase transition in dynamics balances info storage and transfer
  - This is not necessarily located at the topological small-world transition though

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Small-world networks  Information transfer

\[ \langle K \rangle = 4, \ r = 0.36 \]

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Small-world networks Information transfer

$\langle K \rangle = 4$

- Maximisation of coherent information transfer on chaotic side of small-world transition
  - Similar result to random networks: too many interactions cannibalise coherent effect of one source on a destination
Small-world networks  Information transfer

$\langle K \rangle = 4$
Conclusion

Small-world info dynamics

We have observed the small-world topological phase transition to trigger a phase transition in dynamics of RBNs:

- Info storage dominates the ordered regime
- Info transfer dominates the chaotic regime: transfer entropy is maximised near the phase transition
- Small-world networks have a propensity to “combine” comparably large storage and transfer.

We suggest:

- Clustering promotes information storage
- Long links promote information transfer

New empirical and analytic results on a local level (in preparation ...)


Literature


Thank you for your attention!

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